SIMULATION, DESIGN, AND CHARACTERIZATION OF MICROEDDY HYDRODYNAMIC TWEEZERS

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ABSTRACT

Hydrodynamic tweezers are a category of steady streaming micro-scale flows where particles can be trapped near engineered flow obstructions. Particle trapping is set by the particle size, the obstruction size and arrangement, and the frequency of the oscillating flow that drives steady streaming. Here we explore the behavior of neutrally-buoyant particles using a simple model that balances Stokesian drag with the time-averaged drift force induced by the kinematics of motion in this flow. We show a complex behavior for the trapping force vs. shear stress, with certain frequencies showing bifurcated trapping at either a high or low shear stress state.

KEYWORDS: hydrodynamic tweezers, microeddies, particle trapping, steady streaming, shear stress

INTRODUCTION

Steady streaming flows have been studied for decades with numerous analytical, numerical, and experimental methods. The flow structure of a steady streaming system has a series of eddies superimposed on the harmonically oscillating flow that drives the system [1, 2]. In recent years, steady streaming flows have begun appearing in microfluidic systems for applications tied to cell lysis, cell trapping, and chemical reagent mixing and reaction [3, 4, 5]. A key trait of steady streaming in a microchannel is the nature of the obstruction feature that causes flow to deviate from its otherwise rectilinear character. Our group has demonstrated steady streaming in the vicinity of cylindrical obstructions as a method to trap single-cells; we call this “hydrodynamic tweezers” [6]. For micro-scale features (e.g. cylinders with diameters of 20-250 µm), the fluid oscillation frequencies and amplitudes needed to generate a trapping flow are of the order 1000 Hz and 1-20 µm, respectively. Computer simulations have aided our understanding and development of new micro-scale feature geometries that can modify the steady streaming eddy symmetry and strength [7]. However, microeddy flow structure alone does not determine the trapping force and shear that a particle (or cell) experiences. These forces can be critical in cell viability, so here we analyze the effect of flow oscillation traits on the trapping force and shear stress experienced by a particle in hydrodynamic tweezers.

THEORY AND METHODS

Simulations are performed in COMSOL Multiphysics 3.5a using an analytic-numeric perturbation method that speeds computations of the stream function (ψ) and vorticity (Ω) of the oscillating and steady streaming flow components [8]. The fluid velocity is defined here as

\[ \mathbf{u} = (\partial \tilde{\mathbf{u}}/\partial y, -\partial \tilde{\mathbf{u}}/\partial x) \]

where \( \tilde{\mathbf{u}} = \psi_{\omega} \sin t + \psi_{c} \cos t + \epsilon^2 \psi_n \)

(1)

\[ \tilde{\mathbf{u}} = \frac{\mathbf{u}}{\omega a^2} \]

(2)

\[ \tau = \left[ \left( \frac{\partial^2 \psi_u}{\partial x^2} - \frac{\partial^2 \psi_u}{\partial y^2} \right)^2 + \left( \frac{\partial^2 \psi_u}{\partial x^2} - \frac{\partial^2 \psi_u}{\partial y^2} \right)^2 \right] \]

(3)
RESULTS

In Figure 1, we show both the computed steady streaming flow and experiments in a Lagrangian frame of reference [2]. Fig. 1A shows the simulated stream function for the same oscillating flow conditions presented experimentally in Fig. 1B, where $M = 4.1$. The eddies are counter-rotating and their size is inversely proportional to the oscillation frequency [1, 2]. The experimental particle pathline image in Fig. 1B is revealed by the motion of 1 µm fluorescent beads that track the flow and do not trap ($b/a = 0.04, St = 0.006$). Fig. 1C shows the computed trapping locations for two larger particles ($b/a = 0.2, St = 0.36$). The simulation conditions match the conditions used in Fig. 1D, with $M = 6.4$. In Fig. 1D, 5 µm particles are trapped near a 25 µm cylindrical post. As shown, our analysis of Stokes-dependent large particle trapping accurately matches experimental studies. This includes, for instance, the prediction of trapping off-center of the streaming eddies. More work is required to understand why large Stokes particles trap while small Stokes particles travel with the flow.

![Figure 1: Comparison of simulation and experimental results. (A) Image of steady stream function magnitude in a Lagrangian reference frame computed with $M = 4.1$ is comparable to the experimental image (B) of 1 µm fluorescent particles acquired for ~1 sec with frequency = 4.1 kHz and $a = 25$ µm in water at 25°C. (C) Image of computed trap locations (left traps not shown) with $M = 6.4$ and $b/a=0.2$ is comparable to experimental image (D) of trapped 5 µm polystyrene beads with frequency = 10 kHz and $a = 25$ µm in water at 25°C. X lines in C/D depict 45° from horizontal.](image)

In Figure 2, shear stress and the trapping forces at the predicted trapping location in an eddy were calculated for a single eddy over a range of frequencies from $M = 2$ to $M = 10$. The values charted are dimensionless and normalized by the maximums in each data set. Using the same simulation geometry allows us to compare results from different oscillation frequencies. The graph clearly shows both the shear stress and trapping force increase as the frequency is increased until a maximum is reached. Above a frequency of $M = 10$, the particle is physically excluded by the cylinder (i.e. $b >$ trapping distance). In contrast, with low frequencies eddies become large and the trapping forces become weak. Preliminary experimental studies are in agreement with this relationship, as the trapping of particles at low frequency and low Stokes number is difficult.
**CONCLUSIONS AND IMPLICATIONS**

We have shown that our analytic-numeric approach to simulating steady streaming microeddy flows coupled with our particle trapping analytics is a powerful tool for design of experiments (Figure 1). These simulations provide insight into the flow and trapping characteristics that would otherwise not be easily determined from experimental studies alone. We learn from the simple analysis in Figure 2 of a previously undocumented behavior of hydrodynamic tweezers; certain oscillation frequencies produce complex bifurcated trapping capabilities with either a high or low shear stress state. This characteristic may make possible gentler trapping for shear-sensitive particles or cells, or conversely high shearing environments where these effects are desired. Future experiments and simulations will explore these phenomena in more detail.

**ACKNOWLEDGMENTS**

Partial support was provided by NSF IGERT grant #0654252, NSF GRFP, and the Seattle Chapter of the ARCS Foundation.

**REFERENCES**


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