Educational research

Mapping students' knowledge structure in understanding density, mass percent, molar mass, molar volume and their application in calculations by the use of the knowledge space theory†

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Abstract: Knowledge space theory was used for mapping students' knowledge structures in calculating density, mass-percent, molar mass and molar volume. Data were collected among the 9-10th graders (age 15-16) at two different secondary grammar schools. Students' responses were evaluated in a binary fashion and were used for determining knowledge structures with a systematic trial-and-error process using χ^2 analysis. Based on the students' knowledge structures, the critical learning pathways, the characteristic hierarchies of concepts and the critical concepts were identified and analysed. In students' cognitive structure, molar volume is built on the concept of molar mass. With one group there is a strong connection between the concepts of density, molar mass, molar volume and the calculation of gas volume while with the other group there is no such connection. The reason for this disconnected cognitive structure is the difference in the learning method between the two groups. Students from the second school learned the concepts of density, molar mass, molar volume and mass percent by rote-learning using mnemotechnics. This is a good example that rote learning makes the finding of the connections between concepts hard and gives separated and non-mobilizable knowledge. [*Chem. Educ. Res. Pract.*, 2007, **8** (4), 376-389.]

Keywords: knowledge structure, knowledge space theory, density, mass percent, molar mass, molar volume, empirical study

Introduction

In studying and modelling the cognitive organisation of knowledge we often use graphs and networks. Concept maps can be used for exploring the knowledge structure of individuals, and knowledge space theory as a multidimensional model can be applied for studying the cognitive organisation of knowledge characteristic of a group of students.

Knowledge space theory (KST) was developed in 1982 by Doignon and Falmagne and is described in a book by the same authors (Doignon and Falmagne, 1999). Basic concepts of this theory are: 'knowledge space', 'knowledge state', 'knowledge structure', 'surmise relation' and 'critical learning pathway'. *Knowledge space* defines the knowledge needed to understand a certain subject. In mathematics or science this is defined by a set of problems that a student needs to be able to solve; these problems involve a hierarchical ordering. According to the *surmise relation* if a student is capable of solving a given problem at higher level of the hierarchy, we can surmise that – in ideal conditions – this student can also solve other problems that are at lower level of the hierarchy. In real situations the disturbing effect of the lucky-guess and the careless-error has to be taken into consideration. Each student is

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characterised by a *knowledge state*, which is the summation of the problems the student has solved correctly (for example: [1,3,4] means that the student could solve the problems 1, 3 and 4). A representation of knowledge states for any group of students is called *knowledge structure*. The knowledge structure has to be well graded (e.g. each knowledge state must have a predecessor state and a successor state, except for the null state [0] and the final state with correct answers to all questions [Q]). There are several pathways through the knowledge structure between the null state [0] and the final state [Q]. The most common pathway is called *critical learning pathway*, which is the most probable order in learning concepts. Based on the knowledge structure one can determine the characteristic hierarchy of the knowledge, the most probable hierarchical connectivity of concepts, and the critical concept, the concept that most of the students are ready to learn. 'Knowledge Spaces' by Doignon and Falmagne (1999) presents the formal mathematical details of knowledge space theory.

The application of KST to science concepts has been demonstrated firstly by Taagepera et al. (1997). In their survey, for three concepts (pressure, density and conservation of matter) the same multiple-choice pre-test for all grade level (4^{th} through 12^{th} graders) was administered before the topics were formally taught, and the identical post-test was given afterwards. Using KST analysis they constructed the knowledge structures, and suggested tentative critical learning pathways for each concept. They found that KST is a valuable quantitative assessment method for evaluating student knowledge for two reasons: showing the effectiveness of the classroom teaching experience, and suggesting the most probable learning pathways actually taken by the students.

Later, Taagepera and Noori (2000) used KST to map students' thinking patterns in learning organic chemistry. They defined a knowledge space in organic chemistry based on the electron density distribution as a fundamental organising principle. The comparison of the expert hypothetical critical learning pathway with the novice structure, the most common critical learning pathway deduced from student answers, showed that instead of understanding the structure-reactivity analysis on the basis of electron densities, the students mainly had algorithmic knowledge.

In their third paper Taagepera et al. (2002) used KST for following the development of the bonding concept. Their test consisted of 15 questions in a hierarchical order of difficulty as determined by experts using electron densities as the organising principle. They found that student critical learning pathways differed from the expert pathway in two major areas: the understanding that hydrogen atoms have different electron densities depending on whether they are bonded to oxygen or carbon, and their ability to visualise hydrogen-bonded systems at the sub-microscopic level. Furthermore, KST analysis indicated a weak logic structure in 6 of the 9 students groups. Most of the students seemed to have some disconnected information, which can be easily forgotten.

Arasasingham et al. (2004) used KST to assess student understanding of stoichiometry. They prepared a seven-item test and defined the hypothetical expert learning pathway. Their reasoning was that an understanding of the visual and symbolic representations of individual molecules was important for the understanding of the visual, symbolic, and graphical representations of chemical reactivity, and all these elements were essential in numerical problem solving, conceptualising, and in solving a limiting reagent problem. Comparison of the student critical learning pathways with the expert pathway showed that, contrary to the overall logical connections for the experts (from visualisation, to symbolic representations, to problem solving), students overall thinking patterns were from symbolic representations, to numerical problem solving, to visualisation. This means that acquisition of visualisation skills comes later in the novice knowledge structure, and students can solve numerical problems using memorised algorithms.

Arasasingham et al. (2005) used KST also to assess the effect of web-based learning tools on student understanding of stoichiometry. KST analysis of the pre- and post-tests showed that web-based learning tools improved their understanding, but the critical learning pathways were the same on the pre-tests and the post-tests. This means that in the overall thinking patterns of the students the overall logical connections remained from symbolic representations, to numerical problem solving, to visualisation.

Tóth and Kiss (2006) used KST to explore 13-17 year olds' knowledge in identifying physical composition (pure substance, homogeneous mixture or heterogeneous mixture) and chemical composition (element or compound) of matter, as well as the state of matter (solid, liquid or gas) at the particulate level. Based on the student critical learning pathways, they could not detect long lasting changes in the students' cognitive structure. Only slight and temporary changes could be observed in grade 9 (in identifying the state of matter), and in grade 8 (in identifying physical and chemical composition of matter).

In all these publications cited above authors used KST mainly for constructing and analysing the characteristic knowledge structure of the students' group, suggesting, analysing and comparing students' and experts' critical learning pathways, and analysing the distribution of the students' knowledge states. Besides these outcomes of KST analysis, Tóth et al. (2007) have recently demonstrated additional possibilities. We applied KST to interview data with 1st graders prior knowledge about water. Using a systematic trial-and-error approach, the most probable hierarchical connectivity of concepts, the characteristic hierarchy of the knowledge – fitted best to the original response structure – was found. Based on the expert hierarchy, we could determine the critical knowledge (concept) that most of the students are ready to learn.

This study shows how the KST analysis of the responses can be used for mapping and comparing students' characteristic knowledge structures in understanding and applying basic physical and chemical quantities, e.g. in calculating density, mass percent, molar mass and molar volume, and in calculating density from molar mass and molar volume, as well as in calculating gas volume from mass percent, molar mass and molar volume.

The aim of the study

We used KST analysis to answer the following research questions: is there any similarity or difference between the students' groups from two different secondary schools in the cognitive organisation of the basic concepts, namely

1. in response structure;
2. in characteristic knowledge structure;
3. in the critical learning pathway as the most probable order in learning concepts;
4. in characteristic hierarchy as the most probable hierarchical connectivity of concepts; and
5. in critical concept as the concept that most of the students are ready to learn?

Research methodology

Instruments and subjects

For this study we developed a questionnaire (Appendix) in which students were asked to fill in the empty boxes. To answer the first question students have to know the meaning of density. In the second one, students have to use the relationship between mass percent, mass of solution and mass of solute to answer the question. The third question is connected with the concept of molar mass. The fourth calculation is related to the concept of molar volume. The correct solution of the fifth question needs the knowledge that density can be calculated not only from the mass and the volume, but also from the molar mass and the molar volume. As the relationship $d = M / V_m$ is not usually taught directly, this question may be assigned as a 'problem' type item. In the sixth item students have to calculate the volume of a gas from the mass percent, the molar mass and the molar volume using a given network with empty cells. This question is an 'algorithmic' or 'exercise' type item.

The content validity of the test was checked by the chemistry teachers of the secondary schools (I) and (II). The reliability coefficients (Cronbach-alpha) were found 0.654 and 0.631 for the test in the case of students' group (I) and (II), respectively. These are relatively low values, but one cannot expect better values because (a) the number of the items is small, and (b) this is not a classic homogeneous test, but contains items with differing complexity and difficulty.

Data were collected among the 9-10th graders (age 15-16) at two different Hungarian secondary high schools (I) and (II). The number of students involved this survey was 65 and 57, respectively.

Data analysis

For KST analysis responses were scored in a binary fashion, as they were right (1) or wrong (0). We used Potter's Visual Basic computer program (Potter) for the calculations: for the conversion of response structures into knowledge structures, as well as for finding critical learning pathways, the characteristic hierarchies of the concepts, and the critical items. One of the input files of Potter's software is the binary file (RESP.TXT) containing the response states with its population (Figure 1).

Figure 1. Constructing the first input file (RESP.TXT) for Potter's program from the students' distribution among the different response states (see also Figure 4).

Q1	Q2	Q3	Q4	Q5	Q6	N
0	0	0	0	0	0	5
1	0	0	0	0	0	2
1	1	0	0	0	0	3
1	0	1	0	0	0	7
0	1	1	0	0	0	1
0	0	1	1	0	0	1
1	1	1	0	0	0	14
1	0	1	1	0	0	1
0	1	1	1	0	0	1
1	1	1	1	0	0	9
1	1	1	0	1	0	1
1	1	1	0	0	1	3
1	0	1	1	1	0	1
1	0	1	1	0	1	3
1	1	1	1	1	0	5
1	1	1	1	0	1	4
0	1	1	1	1	1	1
1	1	1	1	1	1	3

```
0 0 0 0 0 0 5
1 0 0 0 0 0 2
1 1 0 0 0 0 3
1 0 1 0 0 0 7
0 1 1 0 0 0 1
0 0 1 1 0 0 1
1 1 1 0 0 0 14
1 0 1 1 0 0 1
0 1 1 1 0 0 1
1 1 1 1 0 0 9
1 1 1 0 1 0 1
1 1 1 0 0 1 3
1 0 1 1 1 0 1
1 0 1 1 0 1 3
1 1 1 1 1 0 5
1 1 1 1 0 1 4
0 1 1 1 1 1 1
1 1 1 1 1 1 3
```

Q1, Q2, Q3 etc. stand for the Questions 1, 2, and 3 etc. respectively, with '1' representing a correct answer and '0' an incorrect answer, and N is the number of students at the same response state

The second input file (KNOW.TXT) contains the assumed knowledge states with the estimated probabilities of lucky-guess and careless error for each item (Figure 2). As shown by the values in the first two rows of this input file, we estimated 10% (0.1) probability for both lucky-guess and careless error.

The Potter's Visual Basic computer program is a simplified version of KST analysis. This program calculates the predicted knowledge state populations, normalise them, and calculate the chi-squared values from the input data. Details are available on the internet (Potter).

Figure 2. Second input file (KNOW.TXT) in Potter's program.

```
0.1 0.1 0.1 0.1 0.1 0.1 -1
0.1 0.1 0.1 0.1 0.1 0.1 -1
0 0 0 0 0 0 0
1 0 0 0 0 0 0
0 1 0 0 0 0 0
0 0 1 0 0 0 0
1 1 0 0 0 0 0
1 0 1 0 0 0 0
0 1 1 0 0 0 0
0 0 1 1 0 0 0
1 1 1 0 0 0 0
1 0 1 1 0 0 0
0 1 1 1 0 0 0
1 1 1 1 0 0 0
1 1 1 0 1 0 0
1 1 1 0 0 1 0
1 1 1 1 1 0 0
1 1 1 1 0 1 0
1 1 1 1 1 1 0
```

The 1st and 2nd rows contain the probabilities of lucky-guess and careless-error for each item (Q1-Q6). The other 17 rows show the knowledge states of the assumed knowledge structure (see also Figure 6) in binary fashion.

In the output file (Figure 3) we can see the knowledge states in the assumed knowledge structure, the calculated probabilities of these knowledge states ('Prob'), the predicted populations ('Pred Pop'), the original populations ('Pop') and the χ^2 value ('Chi Sq') for each knowledge state, and finally the total χ^2 ('ChiSqT'). This total χ^2 together with the degrees of freedom characterise the degree to which the assumed knowledge structure fits to the original response structure. The degrees of freedom (d. f.) can be calculated as follows: d. f. = the number of knowledge states in the knowledge structure + the number of estimated parameters (lucky-guess and careless error) – 1. The numbers appearing on the first column in the output file are the codes of the knowledge states in decimal system.

Figure 3. Output file in Potter's program.

```
n=18   m=17   Population =65

Knol.st.            Prob    Pred Pop     Pop     Chi Sq
    0   000000   0.05818    3.78155        5    0.39259
   32   100000   0.04183    2.71923        2    0.19024
   16   010000   0.01338    0.86948        0    0.86948
    8   001000   0.01948    1.26642        0    1.26642
   48   110000   0.05516    3.58530        3    0.09555
   40   101000   0.09960    6.47404        7    0.04273
   24   011000   0.03282    2.13333        1    0.60208
   12   001100   0.01704    1.10774        1    0.01048
   56   111000   0.18165   11.80719       14    0.40724
   44   101100   0.03964    2.57690        1    0.96496
   28   011100   0.02768    1.79916        1    0.35497
   60   111100   0.13012    8.45779        9    0.03476
   58   111010   0.03729    2.42391        1    0.83647
   57   111001   0.05767    3.74863        3    0.14951
   62   111110   0.07376    4.79426        5    0.00883
   61   111101   0.06784    4.40966        4    0.03806
   63   111111   0.04685    3.04539        3    0.00068
ChisqT(17)= 6.265
```

n: number of initial response states (see also Figure 4);
m: number of knowledge states in the assumed knowledge structure (see also Figure 6);
1st column: code of the knowledge state in decimal system;
2nd column: code of the knowledge state in binary system;
3rd column ('Prob'): the probability of the population in the given response state;
4th column ('Pred Pop'): the predicted (calculated) population in the given response state;
5th column ('Pop'): the (initial) population in the given response state;
6th column ('Chi Sq'): χ^2 calculated from the 'Pop' and 'Pred Pop' values;
ChisqT(17): the total value of χ^2 in case of 17 assumed knowledge states.

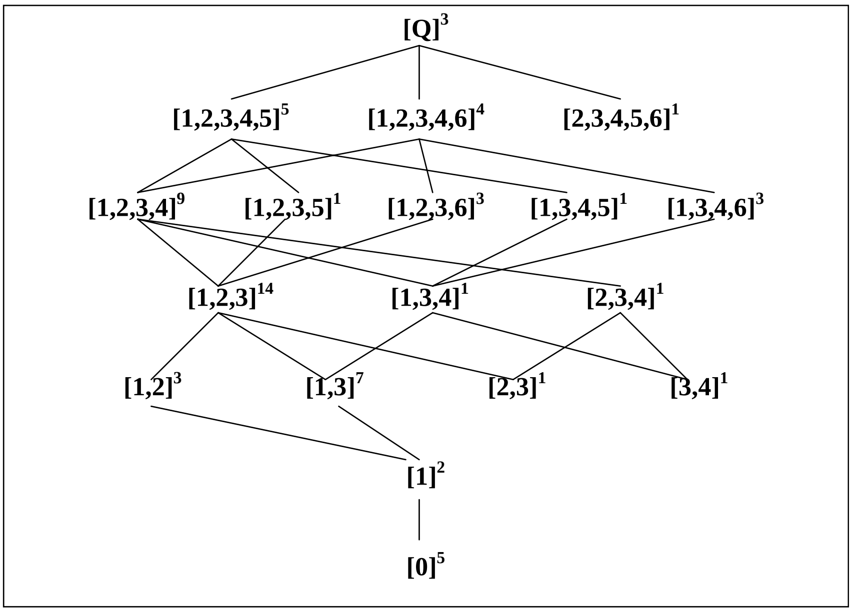
The finding of the knowledge structure that fitted best to the response structure was a systematic trial-and-error process. We started with the most populated response states, then added and subtracted response states to minimise the χ^2 values while forming an interconnected network where each state (except of 0 and Q) had a preceding state and a succeeding state (i.e. the structure was well graded).

In determining the critical learning pathways we also used the Hexagon Data Analysis (hDA) from the lloydesign software developed the University of California at Irvine research group recently (Lloyd). In this method the original input data (response states) are converted into the empirical knowledge structure having all the possible response states with different predicted population. Starting from this empirical knowledge structure hDA gives the proposed knowledge structure and the top four pathways in a few minutes.

Results and discussion

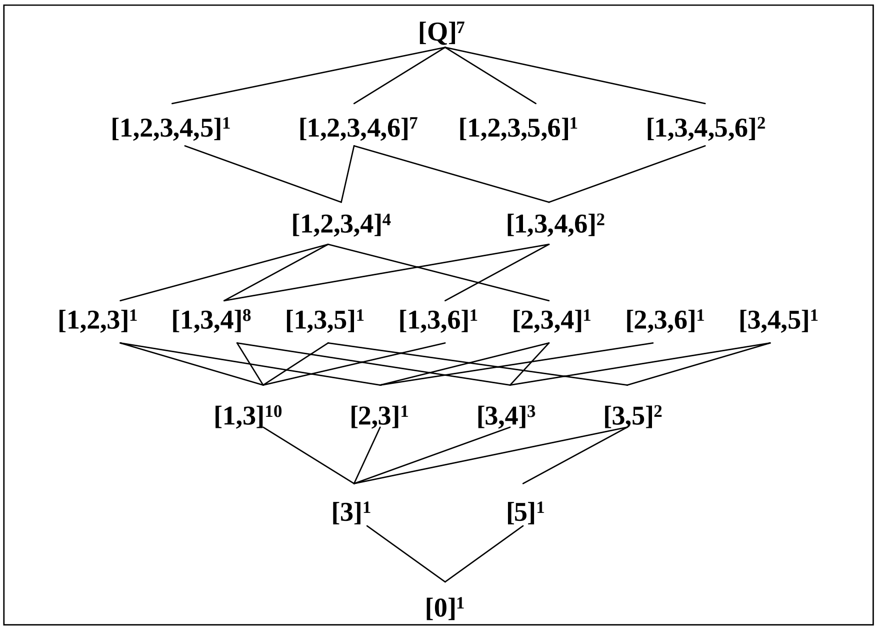
Figure 4 shows the response structure of the students group (I), while Figure 5 represents that of the students group (II). According to the statistical analysis there is significant (p = 0.0053) difference between the response structure of the two students groups.

Figure 4. Response structure of student group (I).



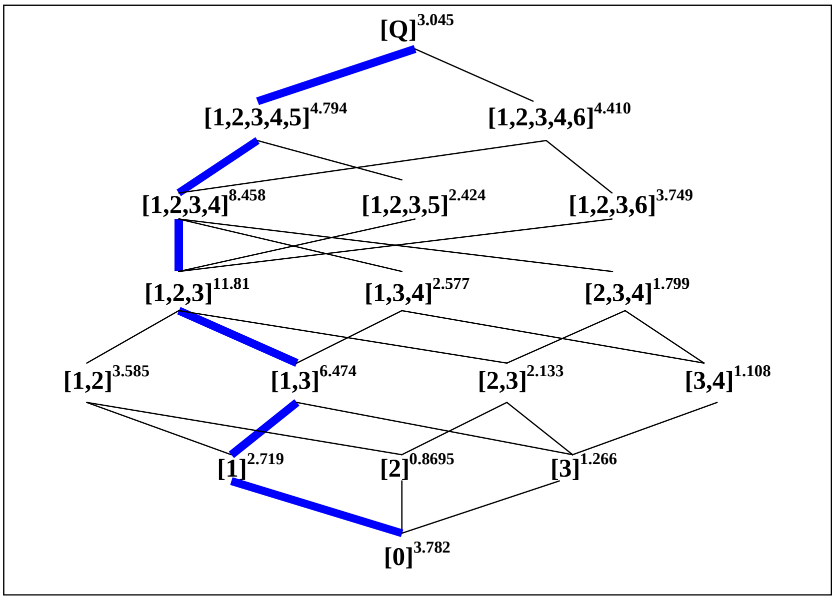
In these response structures, for example, $[Q]^3$ means that only three students gave correct answer for all the questions, $[1,2,3,4]^9$ means that 9 students could solve items 1, 2, 3, and 4 only, and $[0]^5$ means that there were five pupils who could not solve any items at all. Figures 4 and 5 show that the response structures contain only 18 (group (I)), and 21 (group (II)) response states instead of the theoretically feasible 64 (2^6). These figures also show that response structures are not necessary well graded, for example there are no predecessor states for response states [2,3], [3,4], [2,3,4,5,6] (in Figure 4), [1,2,3,5,6] (in Figure 5), and there are no successor states for [1,3,5], [2,3,6], and [3,4,5] (in Figure 5).

Figure 5. Response structure of student group (II).



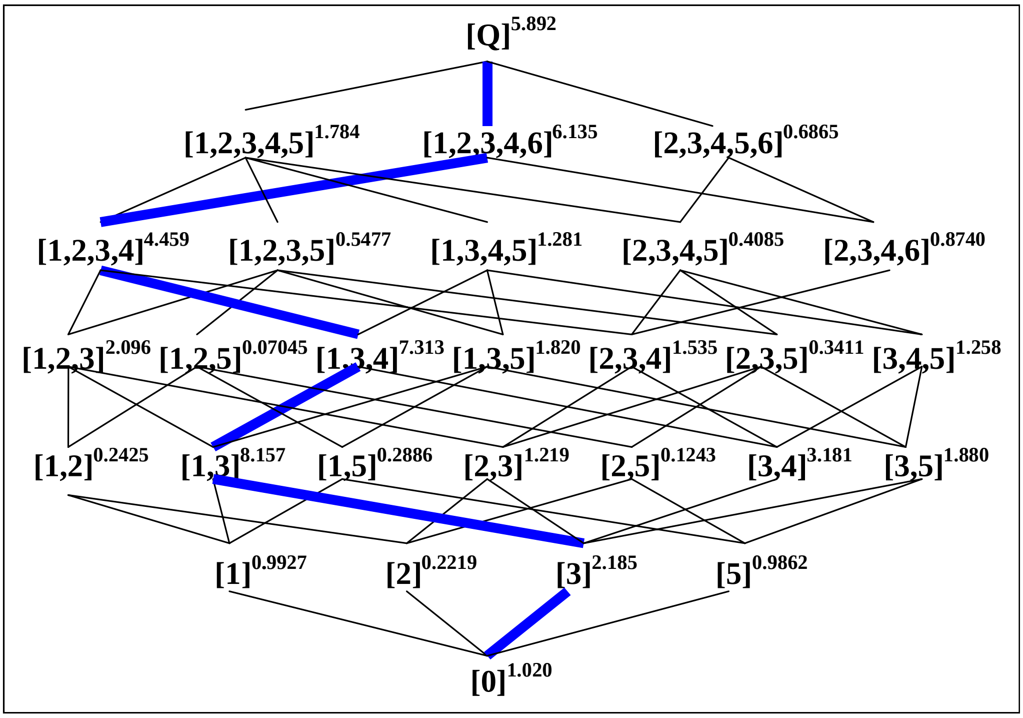
Starting from these response structures, we recognised a subset of response states (the so-called *knowledge structure*) fitted to the original response structure with at least p = 0.05 level of significance. To find the knowledge structure we used Potter's software (Potter), and in fitting process we kept the following in view: *(i)* Lucky-guess and careless-error parameters (0.1 as usual) for each item were estimated. *(ii)* The knowledge structure has to be well graded (*e. g.* each knowledge state must have a predecessor state and a successor state except of the null state [0] and the final state with correct answers to all questions [Q]). The knowledge structures shown in Figures 6 and 7 fitted very well (>99.9%, $p < 0.001$) to the initial response structures. (The calculated 'predicted population' is signed as superscript next to the knowledge states, *e. g.* $[2,3,4]^{1.799}$.) It is seen from these pictures that the knowledge

Figure 6. Knowledge structure of students group (I) (χ^2 = 6,265; df = 28; p<0.001; >99.9%). Critical learning pathway is shown by bold lines.



structure of the students in group (II) contains 28 knowledge states (Figure 7) and is more complicated than that of the students group (I) containing only 17 knowledge states (Figure 6). This difference in the number of knowledge states in knowledge structure indicates that knowledge is less organised in case of the students of group (II) than that of the students of group (I).

Figure 7. Knowledge structure of student group (II) (χ^2 =13.34; df = 39; p <0.001; >99.9%). Critical learning pathway is shown by bold lines.



Among the pathways from the null state [0] to the final state [Q] the most probable pathway (pathway containing knowledge states with the highest product of the populations) was identified as the critical learning pathway characteristic of the students group. Note we used other three methods, too, for determining the critical learning pathway. Among the critical learning pathways obtained from the different methods we selected the one that was the result of three or four of the methods used. Figure 8 shows these critical learning pathways and the learning pathway suggested by the teaching sequence of these concepts and produced by the chemistry teachers. This expert's (teachers') pathway is: density → mass percent → molar mass → molar volume → 'exercise' → 'problem'. It is seen that main differences between these critical learning pathways are in the position of item 2 (mass percent) and item 3 (molar mass). Students learn molar mass after mastering in calculation of mass percent (see expert's pathway). However in the mind of 9th-10th graders molar mass precedes mass percent, and in the case of student group (II) these concepts are situated far from each other in the hierarchy. The inverse position of items 5 and 6 in the critical learning pathways suggests that the students of the secondary school (I) are more familiar with applying density when solving item 5 ('problem') than students from group (II). Students from the school (II) were able to solve the 'exercise' type item more successfully than the 'problem' type item, just as the teachers, (experts) expected. It means that students in the group (II) tend to be algorithmic problem solvers in contrast to students from school (I).

Figure 8. Critical learning pathways for experts and for student groups (I) and (II).

Experts

(1) → (2) → (3) → (4) → (6) → (5)

I)

(1) → (3) → (2) → (4) → (5) → (6)

II)

(3) → (1) → (4) → (2) → (6) → (5)

Using a systematic trial and error process and χ^2 analysis, we determined the hierarchy of the concepts (items) characteristic of the cognitive organisation of the students' knowledge (Figures 9 and 10). We used Hasse diagrams (see for example: Albert and Held, 1994) for the representation of this hierarchy. Accordingly, hierarchy in Figure 9 means, for example, that the knowledge needed to answer item 3 correctly is essential knowledge for items 4, 5, and 6. Knowledge for item 6 is built on the knowledge needed to answer correctly items 2, 3 and 4, but it is independent of the knowledge for items 1 and 5. To solve item 5 students have to have knowledge required for items 1, 3 and 4.

Figure 9. The best model for the organisation of knowledge in students' minds in student group (I) (χ^2 =6.423; df = 28; p <0.001; >99.9%).

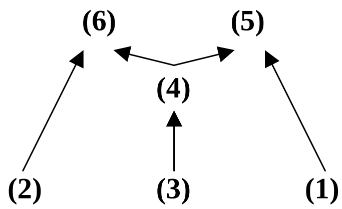


Figure 10. The best model for the organisation of knowledge in students' minds in student group (II) (χ^2 =13.34; df = 39; p <0.001; >99.9%).

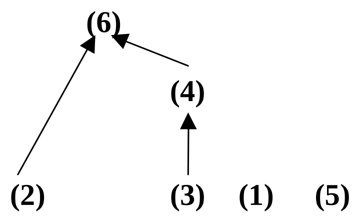
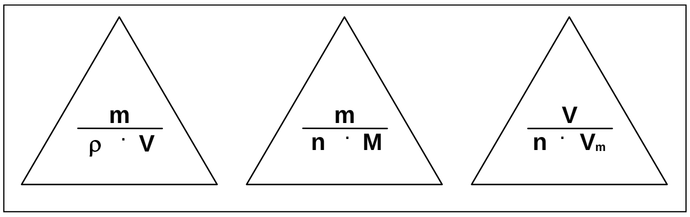


Figure 9 shows the model for describing the organisation of knowledge of the students' of group (I). This model matches the experts' model, and presents clear and logical connections between items.

In contrast, the model obtained for the students' of group (II) (Figure 10) shows a disconnected cognitive structure. In this model item 1 (density with mass and volume) and item 5 (density with molar mass and molar volume) are totally separated from each other, and item 5 is also separated from item 3 (molar mass) and item 4 (molar volume). The probable interpretation is – as seen from the written responses – that students of school (II) learned the concept density, molar mass, molar volume and mass percent mainly by rote, using mnemotechnics presented in Figure 11. However, they did not learn how density could be calculated from the molar mass and molar volume. Rote learning made it difficult for the students to find the connections between the concepts and to apply the learned concepts in solving a new problem.

Figure 11. Mnemotechnics used by students from school (II).



It is interesting that every model in Figure 9 and 10 contains a hierarchical connection between items 3 and 4. This means that in students' cognitive structure the concept of molar volume is built on the concept of molar mass.

Figure 12. Distribution of students among the knowledge states in experts' knowledge structure student group (I).

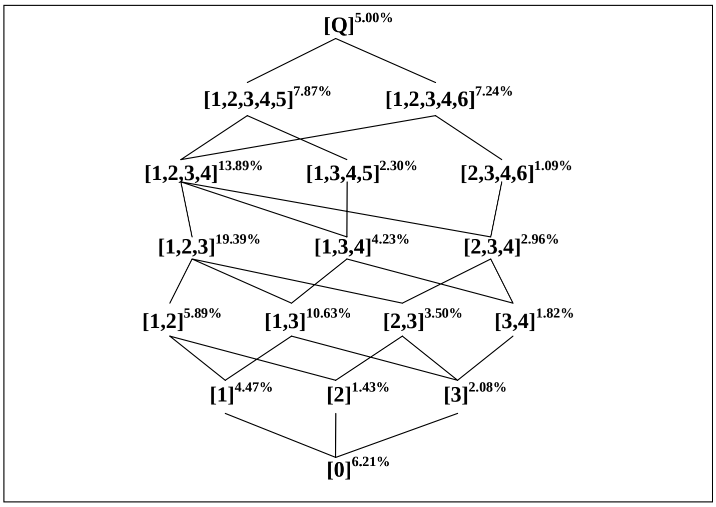
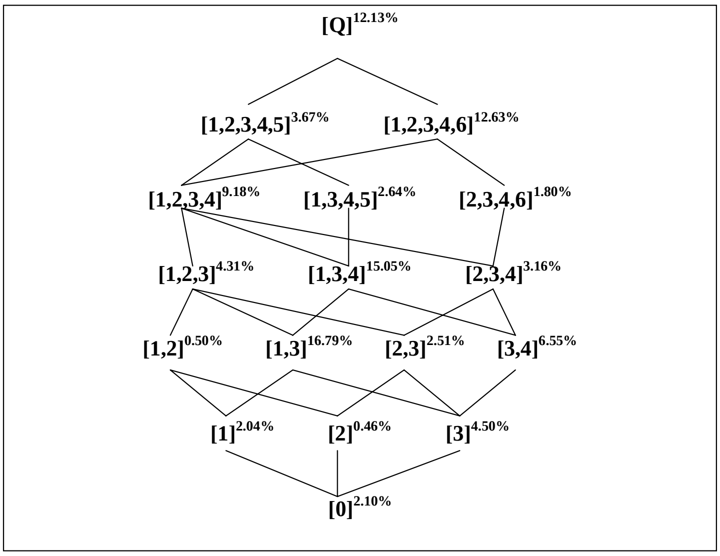


Figure 13. Distribution of students among the knowledge states in experts' knowledge structure student group (II).



Knowledge space theory can be applied not only for studying the knowledge structure of students groups, but also we can use it to optimise the teaching process. If we assume the hypothetical expert hierarchy of items is that shown in Figure 9, we can derive the *hypothetical knowledge structure* indicating the connections between the possible knowledge states (Figures 12 and 13).

Based on the probabilities of the knowledge states in the hypothetical knowledge structure for each student group we can calculate what percentage of students (Table 1) are ready to learn the concept(s) regarding the given item. It is seen that the fitting of the hypothetical knowledge structure to the response structure is very good for each group. This analysis shows that most of the students (35.6%) in group (I) are ready to learn the concept of molar volume (item 4), while most of them (49.7%) in group (II) are ready to learn mass percent (item 2). This means that for students group (I) the molar volume, and for students group (II) the mass percent is the critical concept. Therefore instruction will be the most effective if the teachers discuss molar volume (with group I) and mass percent (with group II), at an early stage.

Table 1. Fitting of experts' knowledge structure to the response structure and percentages of students ready to learn the concepts linked to the given item.

Students' group	Fitting	Item 1	Item 2	Item 3	Item 4	Item 5	Item 6
(I)	99.9%	19.1 %	31.7 %	18.0 %	**35.6 %**	25.4 %	24.7 %
(II)	99.3%	21.1 %	**49.7 %**	5.1 %	28.1 %	36.9 %	19.4 %

Conclusions

The results and conclusions of our study can be summarised as follows.

1. We found significant difference in the characteristic knowledge structures of the student groups from different secondary high schools. The knowledge structure of the students of group (II) is more complex than that of the students of group (I), indicating a less organised knowledge in group (II).
2. There are also differences between the two student groups and between experts and novices (students) in the critical learning pathway as the most probable order in learning concepts. Although Hungarian students learn molar mass after learning mass percent, in the students' minds molar mass precedes mass percent. The reason for this change may be that Hungarian 9^{th} and 10^{th} graders use molar mass more frequently in chemical calculations than mass percent. The inverse position of 'exercise' and 'problem' type items suggests that students from secondary school (II) are more typically algorithmic problem solvers than students from secondary school (I).
3. We could identify the characteristic hierarchies as the most probable models of knowledge structure. We found that in the model best fitted to the response structure of the students of group (II), density and the 'problem' type items are separated from each other and from the molar mass and molar volume. Only the connectivity between mass percent, molar mass, molar volume and 'exercise' type item exists according to the solution network given in item 6. Explanation of this cognitive structure containing separate concepts is given by the written responses of the students. Students from secondary school (II) learned density, molar mass, molar volume and mass percent mainly by rote-learning using mnemotechnics. Therefore, they were not able to apply these concepts in solving a 'problem' type question (item 5), but could use them in solving an 'algorithmic' type question (item 6).
4. Alongside these differences, we could find some similarities between the two groups, as well. Models show that in students' minds the concept of molar volume is built on the concept of molar mass. It is understandable, because molar mass is the first molar

quantity students learn, and molar mass is used more frequently in chemical calculations than molar volume. The connectivity between mass percent, molar mass, molar volume and "exercise" type item in the hierarchy of both groups indicates that students from both schools are able to solve the 'exercise' type questions more easily than the 'problem' type ones.

5. Based on the hypothetical expert hierarchy we could select the critical items for both student groups. It was found that molar volume (in case of student group (I)) and mass percent (in case of student group (II)) were the critical concepts that most of the students were ready to learn.

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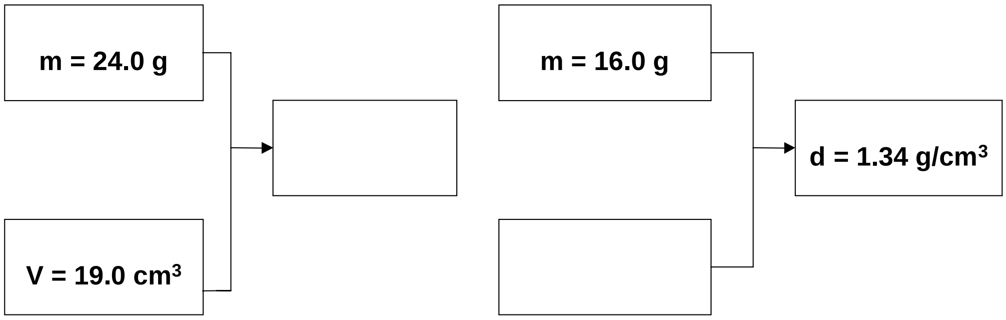
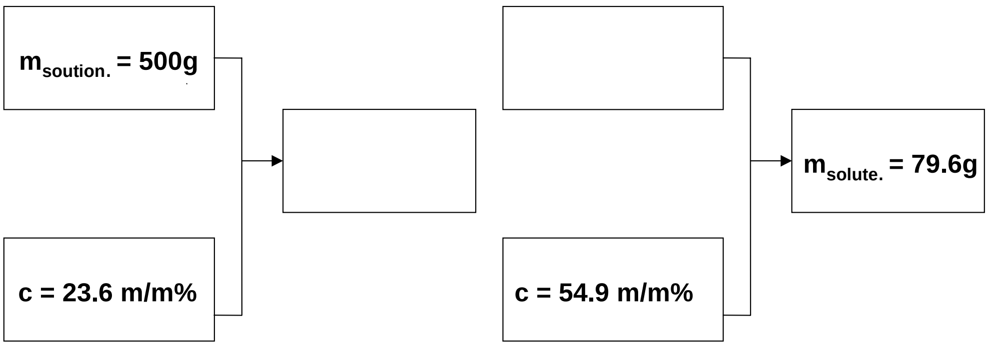
Appendix - Questionnaire

Fill in the empty boxes

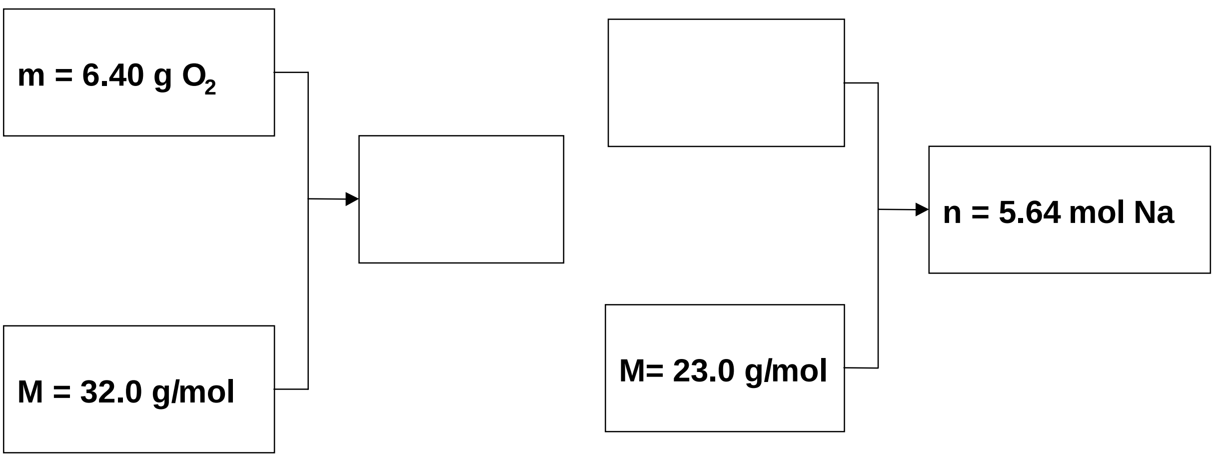
Item 1



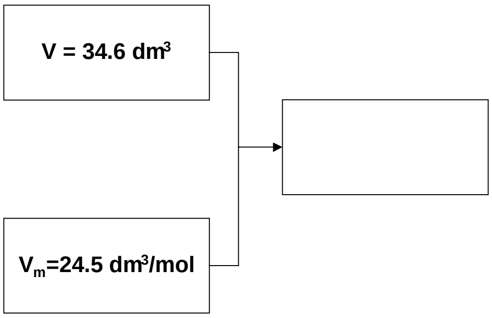
Item 2



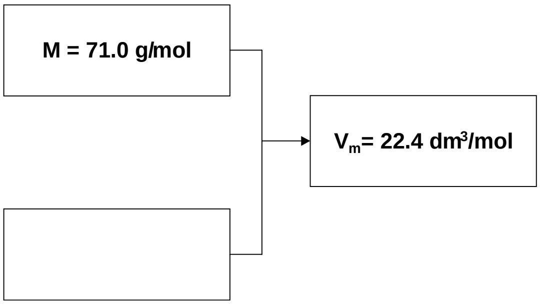
Item 3



Item 4



Item 5



Item 6

How many dm^3 of HCl gas at STP must be dissolved in water to obtain 400 g of 38.0 m/m% hydrochloric acid? M(HCl) = 36.5 g/mol; V_m(HCl) = 24.5 dm^3/mol