Educational research

Explicit teaching of problem categorisation and a preliminary study of its effect on student performance – the case of problems in colligative properties of ideal solutions

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Abstract. Research in science education suggests that the way knowledge is organised affects its availability for problem solving. It also constitutes the important difference between experts and novices. The retrieval of learned schemata from long-term memory is facilitated by categorisation of the problem. In this work, first we suggest a categorisation scheme for problems in the special topic of colligative properties of ideal solutions. Secondly, we report on the results when such a scheme was taught to an experimental group ($n = 41$) of eleventh-grade upper secondary students (age 16-17) in Greece. The group was reduced by considering only students who demonstrated knowledge of the categorisation scheme ($n = 24$), and was compared with a control group ($n = 26$) who were taught in the traditional manner. The experimental group showed a superior performance, but it was not statistically significant. Next, we divided the students into high-, intermediate- and low-achievement subgroups on the basis of their performance in two nationally-examined chemistry courses. No differentiation was found for the students of high and low performance. However, in the intermediate subgroup, students of the experimental group outperformed those of the control group. Because of the limitations (mainly small samples) in the research study, the findings should be treated as preliminary ones. The implications for teaching are discussed. **[***Chem. Educ. Res. Pract.,* 2006**,** 7 (2), 114-130**]**

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Methods and procedures of problem solving

Problem solving is considered an integral component in students' education in science. In school science, problems usually involve for their answer the use of mathematical relationships and the calculation of a numerical result. Such problems contain numerical data, and also the values of physical and chemical quantities and/or constants. Problems on colligative properties of ideal solutions are of this type; they constitute part of the upper secondary school general chemistry curriculum, and are the subject of this paper.

Of great interest is Johnstone's (1993) thorough classification of problems into various types. He has also emphasised a fundamental distinction, that between problems and exercises. A real/novel problem requires that the solver must be able to use what has been termed as higher-order cognitive skills (HOCS) (Zoller, 1993; Zoller and Tsaparlis, 1997; Tsaparlis and Zoller, 2003). As a rule, extensive practice in problems in a particular area can turn problems into exercises. For example, many problems in science can be solved by the application of well-defined procedures (*algorithms*) (Bodner, 1987) that can turn the problems into algorithmic exercises. Problems on colligative properties of solutions are of this type and particularly subject to this transformation. According to the Johnstone classification, in such problems data are given, the method can become familiar, and the outcomes/goals are given. If this is the case, then the problems can be solved by recall and application of algorithms. In this work, we study ways to make effective students' familiarisation with the method of solution. For this purpose, we concentrate our attention on problem categorisation.

In addition to the method, there are a number of other requirements that play important roles in the successful integration of the solution process, leading to the correct result. Relevant to this work are the differences between expert and novice problem solvers and especially: (a) the comprehensive and complete scheme of the experts, in contrast to the sketchy one of the novices; and (b) the extra step of the qualitative analysis taken by the experts before they move into detailed and quantitative means of solution (Simon and Simon, 1978, Larkin, 1980; Reif, 1981, 1983).

In this work, first, we present the theoretical basis for problem categorisation, and then we suggest such a scheme for problems in colligative properties. The scheme was the result of a systematic process. Finally, we report on a *preliminary* investigation of the effect of *teaching explicitly* (that is, offering students an explicit method for choosing between problem solutions) this problem categorisation on upper secondary students' problem-solving ability. The question behind the study is *whether the scheme is helpful to learners in terms of improving overall task performance.* The pressures on this kind of research (particularly in getting suitable samples) sometimes lead to non-ideal structures. This is the reason for the preliminary character of the investigation.

Rationale: Knowledge organisation and problem categorisation

Researchers in cognitive psychology stress that the organisation of knowledge affects the availability and the retrieval process of conceptual schemata during problem solving (Sternberg, 1981; De Jong and Ferguson-Hessler, 1986), and that it is important that a plethora of connections exist among various concepts (Chi and Koeske, 1983). Johnstone (1991) has pointed out that information that is well organised and connected in long-term memory is more easily recalled than specific information, which lacks organisation and connections. Tsaparlis (1994, 1998) has confirmed this in the case of solving very simple (two-step) organic chemical synthesis problems.

Problem categorisation in a specific topic involves organisation of the various problems into a number of categories and subcategories. A solution of a problem begins with a brief analysis of the problem so that it can be categorised mentally. A crucial fact is that experts and novices organise their knowledge differently in a domain. Experts' knowledge networks are characterised by a multilevel structure that is constructed in such a way that they are easily retrieved from long-term memory (Wilson, 1994). Each problem category functions as a cognitive structure that, at least for experienced solvers, includes possible solution methods.

Bunce and Heikkinen (1986) have proposed the explicit method of problem solving (EMPS) which aims to teach novice students the problem-solving analysis procedures used by experts. According to Reif (1981), this analysis helps students *encode* the pertinent information of the problem, which is a major difference in the problem-solving behaviour of experts and novices. *Encoding* is defined by Sternberg (1981) as the identification of each term in the problem, and retrieval from long-term memory of the attributes of these terms that are thought to be relevant to the solution of the problem.

According to Bunce, Gabel, and Samuel (1991), an important part of the encoding process is problem categorisation. If students cannot correctly categorise a problem, they will not be able to retrieve the relevant information from long-term memory. A subsequent step in EMPS leads students to relate the encoded parts of the problem in a schematic diagram of the solution path. After such an analysis, students can use mathematics to reach an algebraic solution and eventually a numerical answer. The above authors Bunce et al. further examined the effectiveness of EMPS and reported that specific instruction in problem categorisation techniques improved achievement scores for combination problems, requiring more than one chemical concept in their solutions, but not for single-concept problems. On the other hand, such training alone was found insufficient to lead to conceptual understanding.

The mechanisms of implicit learning, explored in the cognitive psychology literature, may offer another explanation whereby students become increasingly sensitive to patterns (regularities) in the problem conditions, without ever gaining conscious awareness of the links between conditions and solution process. It is important to note, however, that the research on implicit learning has only investigated simple perceptual rules in experimental psychology settings, as opposed to the learning that takes place in formal education [see Pacton et al. (2001) for a review]. The literature on second language acquisition also investigates the relative advantage of explicitly teaching grammar rules. Truscott (1998) reviewed research in this area and found little evidence to suggest that explicit grammar teaching is helpful in gaining language competence in naturalistic language situations.

Finally, of special importance in problem solving is the *logical structure* of a problem. Niaz (1995) specified this structure by the number of *operative schemata* entering the problem. According to Piaget (Inhelder and Piaget, 1958), a *schema* is an internal structure or representation, while the ways we manipulate schemata are called *operations.* The unit on colligative properties starts with a background sub-unit (A) on the vapour pressure of a single liquid substance (not a colligative property in itself), and is followed by the three sub-units (B-D) on colligative properties as follows:

A. Vapour pressure of a single liquid substance.
B. Vapour pressure depression of an ideal solution of a non-electrolyte.
C. Boiling-point elevation and freezing-point depression of an ideal solution of a non-electrolyte.
D. Osmosis and osmotic pressure of a solution.

The suggested categorisation classifies each problem on the basis of the type of colligative property involved. Each sub-unit involves one category in the scheme. The four categories, plus the sub-categories (see below) in each, constitute the logical structure of these problems.

Method

The problem categorisation scheme

At the start of this research project, the junior investigator worked out an initial categorisation scheme for each sub-unit under study (see above). Work started out with the sub-unit on the vapour pressure of a single liquid substance. Following that, the two investigators discussed and devised a scheme for the categorisation. This process was repeated several times until an agreement was reached. The same process was followed for each of the three colligative properties. The whole process for working out the categorisation schemes was very detailed and systematic, and took several months.

As mentioned above, the suggested categorisation classifies each problem on the basis of the type of colligative property involved, and involves four main groups (categories) of problems. One additional group was employed that contains problems combining two or more colligative properties. The suggested categorisation is shown in outline in Tables 1 and 2, and in full in the Appendix.

Table 1. Outline of the proposed problem categorisation* in the topic of colligative properties of ideal solutions

A. *Vapour pressure of a single liquid substance*	**B.** *Vapour pressure depression of an ideal solution of a non-electrolyte.*	**C.** *Boiling-point elevation and freezing-point depression of an ideal solution of a non-electrolyte.*	**D.** *Osmosis and osmotic pressure of a solution.*
A1. A single liquid substance not in equilibrium state.	**B1.** A non-electrolytic solution of a **non-volatile** substance.**	**C1.** A non-electrolytic solution of a **non-volatile** substance.**	**D1.** A non-electrolytic solution of a non-volatile substance.**
A2. A single liquid substance in equilibrium state.	**B2.** Dilution or concentration of a solution.	**C2.** Dilution or concentration of a solution.	**D2.** Dilution or concentration of a solution.
A3. A single liquid substance in two (or more) independent equilibrium states	**B3.** Mixing of solutions.	**C3.** Mixing of solutions.	**D3.** Mixing of solutions.
	B4. Two (or more) independent solutions at equilibrium state.	**C4.** Two (or more) independent solutions.	**D4.** Two (or more) independent solutions.
	B5. A solution of a **volatile** substance.		**D5.** Two non-electrolytic solutions separated by a semi-permeable membrane.
			D6. Pure solvent and a solution are separated by a semi-permeable membrane.

* Each problem involves just one colligative property.

** Unless stated differently (see Case **B5**), in all other cases, the solute(s) is (are) substance(s) that are non-electrolyte(s) and non-volatile.

Table 2. The case of problems that combine one or more colligative properties.

Problems that combine one or more colligative properties.	
1. Data include concentration. To be found: values of two or more colligative properties.	**2.** Data include value of one colligative property concentration. To be found: values of one or more other colligative properties.

Empirical investigation

The empirical investigation was carried out in the school year 1999-2000 in two public upper experimental secondary schools (*lykeion*) in the Greater Athens region. Experimental schools in Greece, while selecting their students by a draw among the applicants, are schools of higher standards in terms of their teaching staff, and consequently in terms of their student population. Five entire eleventh-grade classes were used, two from school 1 and three from school 2. The students of these classes followed the first out of two years of the 'Positive Stream' of Greek upper secondary school; this stream leads to higher education in medicine and other health related subjects, as well as to science and engineering. The five classes were divided into an experimental group (one class in school 1 and two classes in school 2) and a control group (one class in each school).

The material on colligative properties was the same and taught by the same teacher in each school (one teacher in school 1 and another teacher in school 2). Within each school, the same teacher taught both the experimental and the control group. Both teachers had done graduate studies in chemistry education, and also had long teaching experience.

Method of instruction in the experimental groups

Instruction in the experimental groups started with the theory of sub-unit A with the teaching methodology that the particular teachers usually follow, and within the teaching time that is dictated by the formal national curriculum. Following that, the teachers delivered instruction on the proposed categorisation of the problems in the sub-unit, providing in each case a brief suggestion/methodology for dealing with the corresponding problem. Note that these suggestions/methodologies are not shown in Tables A1-A5 in the Appendix, but two examples are included in the Appendix.

At that time, the students had in front of them a copy of the categorisation scheme that dealt only with the section dealing with the sub-unit under study (this was similar to one of the Tables A1-A5 of the Appendix). This categorisation scheme was prepared by the investigators and distributed by the teacher. Next, the teachers solved one problem on the chalkboard in collaboration with the students, after the students had assigned the given problem to the corresponding category and sub-category. Finally, for practice and consolidation of the methodology, the teachers distributed a limited number (2-3) of additional selected problems, requiring the students to categorise the problems by consulting the categorisation sheet. Students had to hand in both the solved problems and their categorisation at the start of their next chemistry class. The same teaching procedure was employed four more times, for each of the remaining three categories and for the ‘combination problems’.

Method of instruction in the control classes

Instruction in the control classes started with teaching the theory of sub-unit A in the same way as in the experimental classes. Following that, the teachers solved *further* relevant problems on the chalkboard in collaboration with the students. Taking into account that the same teaching time was employed for the experimental and the control groups, as well as the fact that the instruction on problem categorisation took up a significant portion of the available teaching time, it is obvious that in the control group more problems were solved. Again, at the end of the class, the teachers distributed the same limited number (2-3) of additional selected problems for practice at home. Students had to hand in sheets with the solved problems at the start of their next chemistry class. The same teaching procedure was employed four more times, for each of the remaining three categories and for the ‘combination problems’.

Comparison of performance

Students in all experimental and control classes were assessed on their ability to solve problems on colligative properties by administering to them the same test. This constituted the actual formal test in the chemistry course, dictated by the school regulations for the first term, and made a significant contribution to the formal marks for the first term. As a result, students treated the test seriously.

The test contained three problems. Each problem dealt with a different colligative property and involved simple numerical calculations. The problems are given in the Box. To

The three problems of the written test

Problems 1 and 2

The following two solutions, S_1 and S_2 are given:

S_1: A molar solution of substance A ($M_r = 40$) in a solvent X ($M_r = 80$) has a mass of 200 g.

S_2: A molar solution of substance B ($M_r = 90$) in the same solvent X has a mass of 150 g.

Problem 1. If the two solutions S_1 and S_2 have the same boiling point under the same external pressure, calculate the % w/w.

Problem 2. How many grams of solvent should be removed by evaporation from solution S_1 to produce a solution having a vapour pressure of 30 mm Hg?

Data: Vapour pressure of solvent X at the temperature of the experiment $P^o = 37.5$ mm Hg.

Problem 3

The following two solutions, S_3 and S_4 are given:

S_3: A molar solution of substance C ($M_r = 50$) in a solvent Y has a mass of 400 g

S_4: A molar solution of substance D ($M_r = 100$) in the same solvent Y has a mass of 400 g

What is the osmotic pressure at 27°C of solution S_5 that results from mixing the two solutions S_3 and S_4.

Data: (1) Substances C and D do not react chemically with each other.

(2) The density of the solution S_5 is $d = 0.80$ g / mL at 27°C.

(3) The value of the gas constant is $R = 0.082$ atm L / (mol K).

In addition, in the experimental classes, the following additional task was set for each of the three problems:

Find out the category and sub-category (for instance A.1.a) for the problem. (e.g. Problem 1 belongs to category, etc.

NOTE: You can consult the sheet with the problem categorisation.

In the control classes, students had with them and could consult the course textbook. The test sheet stated:

NOTE: You can consult your course textbook.

avoid the possibility of cheating among neighbouring students in class, four different, but equivalent, versions of the problems were used.

In the experimental classes, students had with them and could consult the sheets with the problem categorisation. In addition to solving the problems, these students had to categorise the three problems according to the suggested categorisation. In the control classes, students had with them and could consult the course textbook. Students had 70-75 minutes to solve the problems.

For our analysis, we took into account only those students who answered all three problems: n = 26 for the control group, and n = 41 for the experimental group. This experimental group (*the raw experimental group*) was further reduced by removing the students who *did not demonstrate* knowledge of the taught problem-categorisation scheme. Thus, in the *reduced experimental group* (n = 24), only the students who demonstrated knowledge of problem categorisation were included (as checked through their answers). For this reduction, we removed from the raw sample 2 (out of 13) students of high performance in the two national examinations (see below), 3 (out of 13) students of intermediate performance, and 8 (out of 16) students of low performance. Thus, 61% of the removed students were from those of lower performance. This shows that it was mostly students of lower performance in the national examinations who did not have (or did not demonstrate) knowledge of the problem-categorisation scheme.

Statistical analysis of the data

The small size of the samples, together with the lack of normality in the distributions in many cases dictate that non-parametric statistics should be used, in this case the Mann-Whitney test for independent samples (Cohen and Holliday, 1982, pp. 235-242). The small samples, plus a possible problem with the non-equivalence of the control and the reduced experimental group (see below), force us to treat our findings as preliminary ones. Note that because the chapter on colligative properties is excluded from the taught material of the subsequent years, it was not (and it is still not) possible to repeat the educational experiment to increase sample sizes.

Results and comments

Table 3 compares the mean scores (maximum 100) of the students of the control and experimental groups of our study in the five different levels of written testing (see above). For the experimental group, we report results both for the raw sample, and the *reduced experimental group* (see above). The comparison between the control and the raw experimental group shows very small differences in the scores in the three problems and in the mean score. In addition, the difference in the scores in National Examination 1 (in favour of the control group) is very small, while in National Examination 2, the control group had a superior performance. The latter difference might explain why the control group showed higher scores than the raw experimental group in Problems 1 and 2 and in the mean of the three problems.

Table 3. Performance* of the control and the experimental groups in the two national examinations and the problems of this study. (Maximum score: 100.)

	National exam 1	National exam 2	Problem 1	Problem 2	Problem 3	Mean, Problems 1, 2, 3
Control group (*n* = 26)	86.9 (11.7)	81.2 (12.3)	76.8 (26.2)	56.2 (33.3)	75.6 (28.9)	69.5 (22.1)
Raw experimental group** (*n* = 41)	85.5 (16.0	76.2 (22.7)	69.3 (32.1)	53.6 (36.9)	78.3 (26.2)	67.1 (27.4)
Reduced experimental group*** (*n* = 24)	92.8 (9.4)	85.5 (13.8)	82.4 (23.5)	74.0 (28.5)	89.0 (19.7)	81.8 (19.6)

* Mean scores, with standard deviations in parentheses.
** All participating students of the experimental group who answered all three problems.
*** Only experimental-group students who answered all three problems and also demonstrated knowledge of the problem-categorisation scheme.

Turning to the comparison between the control and the reduced experimental group, first we note that the latter has now higher scores in the two national examinations. This resulted from the fact that most of the weaker students in these examinations were excluded from the experimental group (see above). Performance of the experimental group is now higher by about 6% in Problem 1, by 18% in Problem 2, by 13% in Problem 3, and by 12% in the mean of the three problems. The lower difference in Problem 1 could be attributed to its order, since most students might have spent more time on it. By using the non-parametric Mann-Whitney test for independent samples (Table 4), the differences in Problem 2 and the mean of the three problems are statistically significant near the 95% significance level ($p = 0.05$). Significant (also in favour of the experimental group) is the difference in National Examination 1 at $p = 0.02$. Problem 2, as judged by student performance, proved more difficult, and this might be one reason for its showing a higher effect of the method. By using parametric statistical analysis of covariance with the scores of the two national examinations as covariates, we found that in none of the problems were the differences statistically significant. The *F*-ratio values (with significance levels in parentheses) are as follows: Problem 1, 0.00 (1.00); Problem 2, 1.82 (0.18); Problem 3, 1.08 (0.31); mean of Problems 1, 2, 3: 1.49 (0.23); (We repeat that parametric methods of analysis are not appropriate, as our samples in many cases did not follow the normal distribution.)

Table 4. Non-parametric statistical analysis (Mann-Whitney test) for comparisons of control ($n = 26$) versus reduced* experimental group ($n = 24$): large sample** test statistic *Z*, with two-tailed probability of equaling or exceeding *Z* in parentheses.

National exam 1	National exam 2	Problem 1	Problem 2	Problem 3	Mean, Problems 1, 2, 3
2.28 (0.023)	1.44 (0.15)	0.88 (0.38)	1.98 (0.048)	1.70 (0.09)	2.00 (0.045)

* Experimental-group students who demonstrated knowledge of the problem-categorisation scheme.

** The larger of the two samples (here both samples) has more than 20 data points (Cohen and Holliday, 1982, p. 239).

Comparison according to level of performance in the national examinations

The scores in the two national examinations provided a reliable means for dividing the students of our sample into three levels of performance (high, intermediate, and low). This division is independent of their performance in the tests of this study.

Table 5 shows the scores of the control group and the reduced experimental group in the five levels of testing for students of high, intermediate, and low performance respectively in the two national examinations. The criteria for this division are given in the tables. Because of the small number of students, these criteria were set to create groups that were comparable in size. In all cases, students of high performance outperformed students of intermediate performance in all three problems; similarly, the latter students outperformed the students of low ability.

The stability of the scores of the high-performing students is remarkable. These students achieved high scores in all problems, and consequently the taught problem-categorisation scheme had not affected them. At the other end, low-performing students also showed a lack of effect. The significant difference was due to the intermediate students of both the raw and the reduced experimental groups; although they fell a bit behind in (by about 4%) in Problem 1, in the other two problems and in the mean of the three problems, they scored considerably higher than the students of the control group. By using the non-parametric Mann-Whitney test for independent samples (Table 6), only the differences in Problem 2 are statistically significant (the corresponding values of statistic *U* are smaller than the critical values at the 5% significance level). Also, near significance is the difference in the mean of the three problems for the reduced experimental group. Again, Problem 2 showed a convincing effect of the categorisation. Note however the superiority of the reduced experimental group in National Examination 1 (which is near statistical significance).

Table 5. Performance* of the control group and the reduced experimental group in the two national examinations and the problems of this study for the students with *high, intermediate, and low performance*** in the two national examinations.

	National exam 1	National exam 2	Problem 1	Problem 2	Problem 3	Mean, Problems 1, 2, 3
High-performed students						
Control group ($n = 7$)	97.4 (3.4)	96.5 (2.2)	92.4 (8.1)	93.9 (16.2)	95.9 (8.1)	94.1 (6.1)
Reduced experimental group ($n = 11$)	98.3 (2.0)	95.2 (3.1)	93.8 (9.7)	87.3 (18.3)	93.8 (10.1)	91.7 (9.7)
Intermediate students						
Control group ($n = 11$)	89.0 (4.0)	83.0 (5.1)	82.1 (23.8)	45.8 (24.9)	72.7 (32.1)	65.8 (17.5)
Reduced experimental group ($n = 9$)	93.4 (5.4)	84.2 (5.2)	78.4 (27.7)	73.4 (26.5)	93.7 (11.9)	81.8 (17.8)
Low-performed students						
Control group ($n = 8$)	73.8 (11.6)	68.0 (8.1)	60.3 (32.1)	37.5 (29.3)	61.6 (28.3)	53.1 (18.4)
Reduced experimental group ($n = 4$)	76.4 (10.8)	61.6 (15.4)	59.8 (25.8)	38.4 (29.6)	65.2 (36.2)	54.4 (20.6)

* Mean values (%), with standard deviations in parentheses.

** For high-performed students, scores ≥ 92.5%; for intermediate students, 80% < scores < 92%); and for low-performed students 80% ≥ scores.

Table 6. Students of intermediate performance in the two national examinations: non-parametric statistical analysis (Mann-Whitney test) for comparisons of control and experimental groups (values of statistic *U*).*

	National exam 1	National exam 2	Problem 1	Problem 2	Problem 3	Mean, Problems 1, 2, 3
Control group ($n = 11$) versus raw experimental group ($n = 12$).	40	46	62.5	32.5	50	43
	Critical value of *U* for two-tailed test ($p = 0.05$) = 33					
Control group ($n = 11$) versus reduced** experimental group ($n = 9$).	25.5	35	49	21.5	29	24
	Critical value of *U* for two-tailed test ($p = 0.05$) = 23					

* The smaller of the two *U* values is given (Cohen and Holliday, 1982, p. 237).

** Experimental-group students who demonstrated knowledge of the problem-categorisation scheme.

The solution process

The kinds of errors committed

An interesting aspect of this work are the kinds of errors committed by the students, irrespective of the group (experimental or control) they were in. Of 141 main errors that were spotted 84 (59.6%) were due to deficiencies in understanding of fundamental concepts related to solutions (concentration, dilution and condensation, mixing of solutions). One explanation might be that these concepts were last to be taught during the previous (tenth) grade (taught in a hurry or not at all, and possibly also reviewed hastily at beginning of eleventh grade). An additional 18.4% (26 errors) were attributed to problems in understanding of colligative properties and their laws. The remaining 22.0% (31 errors) were due to lack of attentiveness, errors in numerical calculations, and various other reasons.

Numerical calculations in problem solving

Another interesting finding is the frequency of numerical errors committed by the students who had applied the correct solution procedure, irrespective of the group they were in. For this purpose, we identified all students who had applied the correct solution procedure, irrespective of whether or not they performed successfully the numerical calculations involved. There were 44 such students. Of these, 19 (43.2%) found the correct numerical result, while 25 (56.8%) made mistakes in calculations. This demonstrates that a considerable proportion of students fail to perform numerical calculations correctly, a finding that might be attributed to the fact that in their training students are not encouraged to finish the solution of problems that are given to them for practice both at school and home. Instead the emphasis is for practicing in doing as many problems/exercises as possible in the time available.

Concentrating on the way of performing calculations, some of them performed the calculations in a classical/serial manner, while others chose to use simplifications that speed up the calculations. Table 7 summarises these findings.

Table 7. Numerical errors committed by the students while performing numerical calculations by making or not making simplifications.

Number of students who *did not* make numerical errors: 25	Without simplifications: 11 (44.0%)
	With simplifications: 14 (56.0%)
Number of students who made numerical errors: 19	Without simplifications: 16 (84.2 %)
	With simplifications: 3 (15.8%)

Conclusions and implications for teaching and learning

In general, the results of this (preliminary) study did not reveal significant differences in performance between students who were taught problem categorisation explicitly and the control group. Problem categorisation is not an integral part of science teaching, so students do not have the experience or even a positive disposition toward using it. On the other hand, one can put the blame on the serious conceptual difficulties that the students of our study experienced in dealing with colligative properties themselves, as well as with the auxiliary concepts of solution chemistry (concentration, dilution and condensation, mixing of solutions). As a matter of fact, these weaknesses were responsible for 78% of the errors committed, so they left little space for the effect of the categorisation to be felt. Therefore, categorisation techniques are not by themselves capable or sufficient for conceptual understanding (Bunce et al., 1991).

A definite positive argument in favour of the categorisation scheme derives from the methodology used in this study. The experimental group had the sheets with the categorisation scheme and they had an extra task to be completed in the same time as the control group has for just the test. In addition, the control group had the course text available for consultation. From the test scores, it is fair to conclude that training in categorisation is at least equivalent to having more practice at doing these exercises (during the teaching), and the availability of the scheme in written form is at least the equivalent of, and for some students clearly better than, having the course text in terms of the help it can offer.

The most interesting finding resulted from looking separately at the performance of students of high, low and intermediate performance, as these three levels derived independently from their scores in the national examinations in chemistry. Students of high performance were good at solving problems irrespective of whether or not they were taught categorisation schemes. As a result, the methodology had no observable effect on them. These students had constructed on their own mental representations of these problems, resulting in the proper categorisation; hence the offered categorisation scheme was redundant to them. In addition, these students, being more industrious and attentive, should have had more practice in similar problems. At the other end, students of low performance also failed to benefit from the methodology offered. There are many possible reasons for this failure: insufficient conceptual learning, deficiencies in the proper understanding of the problem statements, weakness in manipulating the mathematical relations entering the problems, lack of interest in the particular lesson and/or the whole school process.

There was, however, a group of students who might have benefited from the taught categorisation. It was the group of students with intermediate performance. Students of intermediate abilities and overall performance are more likely to be receptive to the proposed problem categorisation. The above finding suggests that it is useful for teachers to follow a systematic organised approach to teaching problem solving. Problem categorisation, if applied in many different areas of problems, can result in a sound construction of a knowledge base that can contribute to successful problem solving, especially for intermediate-ability and performance students.

Note also that, while in this study, for reasons of economy in time, an explicit method of problem categorisation was used, that is, a receptive/passive model of teaching was adopted, consistent with Ausubel’s meaningful learning theory (Ausubel, 1968), it can be predicted that the suggested methodology can be more successful if an active/constructivist model is adopted. To this end, students can work out the categorisation schemes on their own or collaboratively in groups under the guidance of their tutor. In any case, to become good problem solvers, students must be given ample opportunity to do it. This will not give them just practice, it will develop confidence. Further, students who have obtained good answers should not be ignored by the teacher; they also need guidance and help.

A problem-categorisation scheme essentially constitutes a set of rules that serve to make explicit the conditions in a problem that indicate the solution procedure. Although the assertion that explicit teaching will be helpful appears intuitive, further analysis and a review of the literature point variously to reasons either to support or question this claim. Intuitively, it might be expected that teaching students the link between the conditions and the solution would encourage this link to be made. Whether this link is likely to be invoked in real problem-solving situations remains an open question, but if it is learned, at least the possibility is there! However, it should be noted that at least some students are able to invoke the correct procedure without such teaching, and some probably do so without awareness of the link between conditions and procedure. Some learners may consciously notice and understand the pattern in the conditions, linking this to a particular solution procedure. On the other hand, one should take into account that the teaching of explicit rules adds a further layer of abstraction and complexity that students must process, and such additional information is liable to strain working-memory resources.

The main aim of research on problem solving in science is the development of a theory or models that can explain the interaction between a problem and the problem solver. The usefulness of such models is not simply their explanatory power, but mainly their predictive power. We hope that this study has contributed toward that aim, and that together with previous studies (Bunce et al., 1991) support the belief that explicit instruction in problem solving strategies can increase problem solving ability (Reif, 1981). It can also teach experts' methodology. Because of the errors likely to have occurred (due to the sampling and the 'trimming' of the sample to get the reduced experimental group), the conclusions are only preliminary, so a lot remains to be investigated further.

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Appendix:
The proposed categorisation scheme of problems in colligative properties of ideal solutions

Part 1: Tables setting out the problem categorisation

NOTE: These are the tables that were used by the students.

Table A1. Proposed problem categorisation in the topic of vapour pressure of a single substance.

A. *Vapour pressure of a single liquid substance.*	
A1. *A single liquid substance not in equilibrium state.*	**a.** A quantity of **liquid substance** is placed in an empty vessel at a given temperature. **No** dynamic **vapour-liquid equilibrium** is established. (The amount of the substance is inadequate, so that the liquid completely evaporates P.) P_{vapour} is the vapour pressure.
	b. A quantity of **gaseous substance (vapour of the substance)** is placed in an empty vessel at a given temperature. **No** dynamic **vapour-liquid equilibrium** is established. (The amount of the substance is inadequate, so that all the substance remains in the gaseous state.) P_{vapour} is the vapour pressure.
A2. *A single liquid substance in equilibrium state.*	**a.** A quantity of **liquid substance** is placed in an empty vessel at a given temperature. Dynamic **vapour-liquid equilibrium** is established. *The* vapour pressure is P^{o}.
	b. A quantity of **gaseous substance (vapour of the substance)** is placed in an empty vessel at a given temperature. Dynamic **vapour-liquid equilibrium** is established. The vapour pressure is P^{o}.
A3. *A single liquid substance in two (or more) independent equilibrium states.*	A quantity of the same **liquid substance** is placed in two (or more) empty vessels. Dynamic **vapour-liquid equilibrium** is established ($P = P^{o}$) or not established in each vessel at the same temperature ($P < P^{o}$).

Table A2. Proposed problem categorisation in the topic of vapour pressure depression of an ideal solution of a non-electrolyte.

B. *Vapour pressure* depression *of an ideal solution of a non-electrolyte.*	
B1. *A non-electrolytic solution of a* ***non-volatile*** *substance.**	A quantity of a solution of a substance which is not an electrolyte* (or more than one such substances that do not react chemically with each other) is introduced into an empty vessel at a given temperature. Dynamic **vapour-liquid equilibrium** is established. The vapour pressure is P.
B2. *Dilution or concentration of a solution.*	A quantity of a solution of a substance (or more than one substance that do not react chemically with each other) is **diluted** or **concentrated** at *constant temperature* (by *adding* or *removing* a quantity of solvent). Dynamic **vapour-liquid equilibrium** is established.
B3. *Mixing of solutions.*	A quantity of a solution of a substance (or more than one substance that do not react chemically with each other) is **mixed** with a quantity of *another solution* (or more solutions) at *constant temperature.* Dynamic **vapour-liquid equilibrium** is established.
B4. *Two (or more) independent solutions at equilibrium state.*	Quantities of two (or more) independent solutions of a substance (or more than one substance that do not react chemically with each other), in the same solvent and at the same temperature, are in state of dynamic **vapour-liquid equilibrium**.
B5. *A solution of a* ***volatile*** *substance.*	A quantity of a solution of a **volatile** substance (or more than one **volatile** substance that do not react chemically with each other) is introduced into an empty vessel, at a given temperature. Dynamic **vapour-liquid equilibrium** is established. The vapour pressure is P_{total}.

* Unless stated differently (see Case **B5**), in all other cases, the solute(s) is (are) substance(s) that are non-electrolyte(s) and non-volatile.

Table A3. Proposed problem categorisation in the topic of boiling-point elevation and freezing-point depression of an ideal solution of a non-electrolyte.

C. *Boiling-point elevation and freezing-point depression of an ideal solution of a non-electrolyte.*	
C1. *A non-electrolytic solution of a* ***non-volatile*** *substance.**	A quantity of a solution of a substance which is not an electrolyte* (or more than one such substances that do not react chemically with each other) boils or freezes at a given temperature.
C2. *Dilution or concentration of a solution.*	A quantity of a solution of a substance (or more than one substance that do not react chemically with each other) is **diluted** or **concentrated** at *constant external pressure* (by *adding* or *removing* a quantity of solvent).
C3. *Mixing of solutions.*	A quantity of a solution of a substance is **mixed** with a quantity of *another solution* (or more solutions) of the same substance (or a different substance that does not react chemically with the other substance) at *constant external pressure.*
C4. *Two (or more) independent solutions.*	Two (or more) independent solutions of a substance (or more than one substance that do not react chemically with each other) in the same solvent and at the same temperature. Each of these solutions boils or freezes at a given temperature (different for each one of them).

* In all cases, the solute(s) is (are) substance(s) that is (are) non-electrolyte(s) and non-volatile.

Table A4. Proposed problem categorisation in the topic of *osmosis and osmotic pressure.*

D. *Osmosis and osmotic pressure of a solution.**	
D1. *A non-electrolytic solution of a non-volatile substance.***	A given quantity of a solution of a non-volatile substance, which also is non-electrolyte* (or even more than one such substances that do not react chemically with each other), has osmotic Π, at a given temperature.
D2. *Dilution or concentration of a solution.*	A given quantity of a solution of a substance (or more than one substance that do not react chemically with each other) is **diluted** or **concentrated**, at a given temperature.
D3. *Mixing of solutions.*	A quantity of a solution of a substance is **mixed** with a quantity of *another solution* (or more solutions) of the same substance (or a different substance that does not react chemically with the other substance) at a given temperature.
D4. *Two (or more) independent solutions.*	Two (or more) independent solutions of a substance (or more than one substance that do not react chemically with each other) have osmotic pressures Π_1, Π_2 respectively, at the same temperature (or different temperatures).
D5. *Two non-electrolytic solutions separated by a semi-permeable membrane.*	Two different solutions of the same substance (or more substances that do not react chemically with each other) in the same solvent are separated by a semi-permeable membrane, at a given temperature. The phenomenon of osmosis occurs until a dynamic equilibrium is established.
D6. *Pure solvent and a solution are separated by a semi-permeable membrane.*	Pure solvent and a solution of a substance in the same solvent (or more than one substance that do not react chemically with each other) are separated by a semi-permeable membrane, at a given temperature. The phenomenon of osmosis occurs until a dynamic equilibrium is established. At equilibrium, the two columns of liquids have a difference in height, corresponding to the osmotic pressure of the solutions.

* In all cases, the processes are taking place under constant external (atmospheric) pressure.

** In all cases, the solute(s) is (are) substance(s) that are non-electrolyte(s) and non-volatile.

Table A5. The case of problems that combine one or more colligative properties.

Problems that combine one or more colligative properties.	
1. *Data include concentration. To be found: values of two or more colligative properties.*	Problem involves a solution of a non-volatile substance which also is non-electrolyte (or more than one such substances that do not react chemically with each other). The concentration of the solution in given, conditions are given. Values of two or more colligative properties of the solution are to be found.
2. *Data include value of one colligative property concentration. To be found: values of one or more other colligative properties.*	Problem involves a solution of a non-volatile substance which also is non-electrolyte (or more than one such substances that do not react chemically with each other). The value of one particular colligative property of the solution is given. Values of two or more other colligative properties are to be found.

Part 2. Examples of suggestions/methodology for dealing with the problems

EXAMPLE 1, CASE **A.** *Vapour pressure of a single liquid substance* **(A1)** *A single liquid substance not in equilibrium state***.**

(A1.a) A quantity of **liquid substance** is placed in an empty vessel at a given temperature. **No** dynamic **vapour-liquid equilibrium** is established. (The amount of the substance is inadequate, so that the liquid completely evaporates). P_{vapour} is the vapour pressure at the given temperature.

√ Apply the ideal-gas equation for the vapour:

$$P_{\text{vapour}}\, V_{\text{vapour}} = n_{\text{vapour}}\, R\, T$$

(N.B. $P_{\text{vapour}} < P^{o}$)

EXAMPLE 2, CASE **D**. *Osmosis and osmotic pressure of a solution;* **(D2)** *Dilution or concentration of a solution.*

A given quantity of a solution of a substance (or more than one substances that do not react chemically with each other) is **diluted** or **concentrated** under constant external pressure, at a given temperature.

√ Apply Vant's Hoff's law both for the initial and the final solution:

$$\Pi_1\, V_{\text{solution, 1}} = n_{\text{solute, 1}}\, R\, T$$

$$\Pi_2\, V_{\text{solution, 2}} = n_{\text{solute, 2}}\, R\, T$$

N.B. (a) in dilution: $n_{\text{solute, 1}} = n_{\text{solute, 2}}$

(b) in condensation with evaporation of solvent: $n_{\text{solute, 1}} = n_{\text{solute, 2}}$

(c) in condensation with addition of solute: $n_{\text{solute, 1}} + n_{\text{solute, added}} = n_{\text{solute, 2}}$