Half-lives

The decay of radioactivity is entirely random, and the rate of the process is proportional to the number of radioactive atoms present.

For a beaker containing 10,000 radioactive atoms, these atoms decay 10 times faster than a beaker containing 1,000 atoms.

If we say it takes 1 minute for 500 atoms to decay in the latter case, then it only takes 6 seconds for 500 atoms to decay in the first case, 10 times faster. Therefore, it takes 10 x 6 seconds = 1 minute for 5,000 atoms to decay in the first case.

In both cases, it takes 1 minute for the number of atoms to decay to exactly half. This is the half-life.

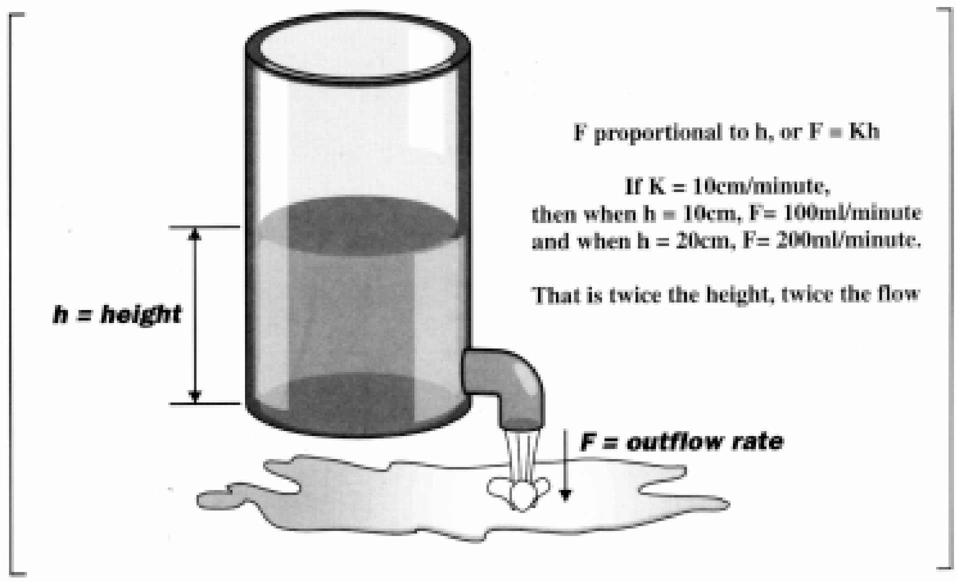
This argument holds whatever number of atoms is considered.

In this case we arbitrarily chose 1 minute as the half-life. For a given radioactive species (isotope) whose atoms are decaying by the emission of alpha, beta or gamma, the time taken for any number of atoms to decay to half, the half-life, is a unique number. The half-life can be used to qualitatively identify a specific isotope because the number is unique.

Analogy

A cylindrical vessel holding water, say, in which the outlflow rate is not controlled by the size of the outflow pipe, but is proportional to the height of the water in the vessel:

Assuming the vessel holds 200 ml when h =10 cm, then it holds 400 ml when h = 20 cm. The half-life for h =10 cm and h = 20 cm is the same; that is the time for exactly half the contents to flow out. For h = 10 cm this is (200/2)/100 = 1 minute; for h = 20 cm this is (400/2)/200 = 1 minute. Therefore the half-life for the outflow of water from this vessel is 1 minute.



Radioactivity values of the half-life vary more than any other measurement known to man. These values range from 10^{13} years (longer half-lives are considered to be stable atoms!) to about 10^{-15} years, which is roughly 10^{-8} seconds, a range of 28 orders of magnitude.

Radioactivity is an excellent example of a first order reaction where the rate is proportional to the number of atoms present.

Mathematics of First Order Reactions

This is for those interested and can be by-passed by the non-mathematical.

If N is the number of radioactive atoms, then the rate of decay is -dN/dt, and this is proportional to the number of atoms N, therefore:

$$-dN/dt = constant \times N \quad ....(1)$$

The constant is called the decay constant and is usually represented by the symbol λ, and

$$-dN/dt = \lambda N, \text{ or } dN/N = -\lambda dt \quad ....(2)$$

On integration, $\ln(N) = -\lambda t + x$ (integration constant) when $N = No$, $t = 0$ and $x = \ln(No)$.

Therefore,

$$\ln(N/No) = -\lambda t, \text{ or } N/No = e^{-\lambda t} \quad ....(3)$$

When $t = t(1/2)$, then $N = No/2$

Or,

$$\ln(1/2) = -\lambda t(1/2), \text{ and } t(1/2) = \ln 1/\lambda = 0.693/\lambda \quad .... (4)$$

It can be seen that t(1/2) is a constant, for any given radioactive isotope, and is inversely proportional to the decay constant of that isotope.

From (3) above, $N = No\ e^{-\lambda t}$, or $A = Ao\ e^{-\lambda t}$, where A is the activity of a radioactive species at time t and Ao is the initial activity of the same species.

If log A is plotted against t, a straight line will be obtained, if it is a single radioactive species, and the slope will give λ, and hence t(1/2). If there are two components, there will be two straight lines.