

6. This question is about a supernova

The electronic ground state (*i.e.* the lowest electronic state) of a hydrogen atom may be written $1s^1$ indicating that the single electron resides in the $1s$ orbital. If sufficient energy is given to the atom, the electron may be promoted from the $1s$ orbital to a higher energy orbital, such as the $2p$ orbital or the $3p$ orbital.

The energy of an electron in a hydrogen atom (or any ionized atom with nuclear charge Z and with just one electron remaining) is given by the following equation:

$$E_n = -R_H \frac{Z^2}{n^2}$$

The energy of a free, ionized electron is zero; electrons in the atom have lower energy, hence the minus sign.

In the equation, Z is the number of protons in the nucleus ($Z = 1$ for hydrogen); n is the principal quantum number ($n = 1$ for the $1s$ orbital, 2 for the $2s$ and $2p$ orbitals, 3 for the $3s$, $3p$ and $3d$ orbitals, *etc.*); R_H is the *Rydberg constant* equal to the ionization energy of a hydrogen atom ($R_H = 2.179 \times 10^{-18}$ J).

- (a) Calculate the energy of an electron in a $2p$ orbital in an excited hydrogen atom.
- (b) Calculate the energy needed to promote the electron in a hydrogen atom from the $1s$ orbital to the $2p$ orbital.
- (c) Calculate the ionization energy of a helium ion, He^+ .

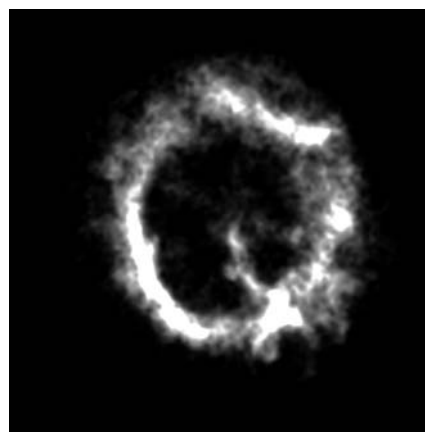
When an electron returns from a higher energy orbital to a lower one, energy is given out as light (the cause of the familiar flame colours). The frequency of the light, f , (in Hz) is related to the energy of the transition, ΔE , by the equation:

$$\Delta E = hf \quad (\text{where } h \text{ is Planck's constant} = 6.626 \times 10^{-34} \text{ J s}).$$

- (d) Calculate the frequency of light for the electronic transition in a hydrogen atom from a $2p$ orbital to the $1s$ orbital (the so-called Hydrogen Lyman- α line).

Supernova remnant E0102-72, located some 200,000 light years away, has been found to contain more than a billion times the amount of oxygen contained in the Earth's oceans and atmosphere. At the incredibly high temperatures in the supernova (many millions of Kelvin), the oxygen atoms are multiply ionized to single electron species, O^{7+} . The oxygen was detected by the specific frequency of its Lyman- α line (the transition $n = 2$ to $n = 1$).

- (e) Calculate the frequency of the O^{7+} Lyman- α line.
- (f) Another element present in large quantities has its Lyman- α line at 2.471×10^{17} Hz. What element is this?



Supernova remnant E0102-72 as photographed by the UV / x-ray telescope *Chandra*.

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$$\begin{aligned}
 \text{(a) } E &= -R_H Z^2 / n^2 \\
 &= -2.179 \times 10^{-18} \times 1/4 \\
 &= -5.45 \times 10^{-19} \text{ J} \quad (1)
 \end{aligned}$$

$$\begin{aligned}
 \text{(b) } E &= -5.45 \times 10^{-19} - (-2.179 \times 10^{-18}) \\
 &= 1.63 \times 10^{-18} \text{ J} \quad (1)
 \end{aligned}$$

$$\begin{aligned}
 \text{(c) I.E.} &= 0 - (-2.179 \times 10^{-18} \times 4/1) \\
 &= 8.72 \times 10^{-18} \text{ J} \quad (1)
 \end{aligned}$$

$$\begin{aligned}
 \text{(d) } f &= 1.63 \times 10^{-18} / 6.626 \times 10^{-34} \\
 &= 2.47 \times 10^{15} \text{ Hz} \quad (1)
 \end{aligned}$$

$$\begin{aligned}
 \text{(e) } ?E &= -R_H Z^2 (1/n_2^2 - 1/n_1^2) \\
 &= -2.179 \times 10^{-18} \times 64 \times (1/4 - 1) \\
 &= 1.05 \times 10^{-16} \text{ J} \\
 f = ?E/h &= 1.05 \times 10^{-16} / 6.626 \times 10^{-34} \\
 &= 1.58 \times 10^{17} \text{ Hz} \quad (3)
 \end{aligned}$$

lose 1 mark for using $Z=7$

(gives $?E = 8.01 \times 10^{-17} \text{ J}$, $f = 1.58 \times 10^{17} \text{ Hz}$)

$$\begin{aligned}
 \text{(f) } ?E &= hf = 6.626 \times 10^{-34} \times 2.471 \times 10^{17} \\
 &= 1.64 \times 10^{-16} \text{ J} \quad (1)
 \end{aligned}$$

$$\begin{aligned}
 ?E &= -R_H Z^2 (1/n_2^2 - 1/n_1^2) \\
 &= -R_H Z^2 (-3/4)
 \end{aligned}$$

$$\begin{aligned}
 Z &= \sqrt{(4/3 \times ?E / R_H)} \\
 &= \sqrt{(4/3 \times 1.63 \times 10^{-16} / 2.179 \times 10^{-18})} \\
 &= 10 \quad \text{The element is Neon} \quad (1)
 \end{aligned}$$

Total 9

Chair's Report

We were very pleased with the way many students attempted Question 6 which involved energy levels in a hydrogen atom. In fact, this question was the surprise of the paper, with a large number of high-quality answers.