Appendix 1

1.1 Rescaled Range Analysis (R/S)

The R/S analysis was introduced by Hurst and attempts to find patterns that might repeat in the future. The method employs two variables, the range, R and the standard deviation, S, of the data. According to the R/S method, a natural record in time, $X(N) = x(1), x(2), ..., x(N)$ is transformed into a new variable $y(n, N)$ in a certain time period $n (n = 1, 2, ..., N)$ from the average, $\langle x(N) \rangle = \frac{1}{N} \sum_{n=1}^{N} x(n)$, over a period of N time units. $y(n, N)$ is called accumulated departure of the natural record in time. The transformation follows the formula:

$$y(n, N) = \sum_{i=1}^{n} (x(i) - \langle x \rangle_{N})$$  \hspace{0.5cm} (1.1)

The rescaled range is calculated from (1.2):

$$R/S = \frac{R(n)}{S(n)}$$  \hspace{0.5cm} (1.2)

The range $R(n)$ in (1.2) is defined as the distance between the minimum and maximum value of $y(n, N)$ by:

$$R(n) = \max_{1 \leq i \leq N} y(n, N) - \min_{1 \leq i \leq N} y(n, N)$$  \hspace{0.5cm} (1.3)

The standard deviation $S(n)$ in (1.2) is calculated by:

$$S(n) = \sqrt{\frac{1}{N} \sum_{n=1}^{N} (x(n) - \langle x \rangle_{N})^2}$$  \hspace{0.5cm} (1.4)

$R/S$ is expected to show a power-law dependence on the bin size $n$

$$\frac{R(n)}{S(n)} = C \cdot n^H$$  \hspace{0.5cm} (1.5)

where $H$ is the Hurst exponent and $C$ is a proportionality constant.

The log transformation of the last equation is a linear relation

$$\log\left(\frac{R(n)}{S(n)}\right) = \log(C) + H \cdot \log(n)$$  \hspace{0.5cm} (1.6)

from which, exponent $H$ can be estimated as the slope of the best fit line.

1.2 The Roughness-Length Method (R-L)

The R-L method is based on the Fractal Geometry concept which is used for the accurate calculation of the Hurst Exponent. The calculation of the Hurst exponent through the R-L method is performed by calculating the standard deviation $S(n)$ of the height values of a segment of length $n$ of a self-affine profile by the formula:

$$S(n) = A \cdot n^H$$  \hspace{0.5cm} (1.7)

In (1.7) $A$ is a proportionality constant that describes the profile waviness amplitude and $H$ is the Hurst exponent. $S(n)$ is calculated by

$$S(n) = \frac{1}{\xi_n} \cdot \sqrt{\frac{1}{m_i-2} \sum_{j<i} (x_j - \langle x \rangle_{m_i})^2}$$  \hspace{0.5cm} (1.8)

where $\xi_n$ is the total number of segments of width $n$ in which the profile is divided, $m_i$ is the number of points included in the $i$-th segment $n_i$, $x_j$ is the aperture of the profile nodes from the best fit line and $\langle x \rangle_{n_i}$ is the mean value of $x_j$ in the segment $n_i$. Representing the pairs of $\{n, S(n)\}$ in a double logarithmic diagram, the Hurst exponent is calculated through a least square fit.

1.3 The Variogram Method

The variogram, also known as variance of the increments, is the expected value of the squared difference between two $x$ values in a trace separated by a distance $h$ i.e. the sample variogram $2\gamma(n, h)$ of a series $x(n)$ is measured by the following equation:

$$2\gamma(n, h) = \frac{1}{M} \sum_{i=1}^{M} [x(n_i) - x(n_i + h)]^2$$  \hspace{0.5cm} (1.9)

where $M$ is the total number of pairs of roughness heights of the profile that are spaced at a lag distance $h$. The variogram $2\gamma(n, h)$ and the Hurst exponent $H$ are related with the equation

$$2\gamma(n, h) = K \cdot h^{2H}$$  \hspace{0.5cm} (1.10)

where $K$ is a proportionality constant. The slope of the linear fit of $\log(2\gamma(n, h))$ and $\log(h)$ equals $2H$.

References