Supplementary information

1. Detailed deduction of exchange energy $E_{\text{ex}}$

The exchange energy $E_{\text{ex}}$ is denoted as the summation of magnetostatic energy between all magnetic moments. In the model of chain of ellipsoid-rings, we only consider the dominant energy of magnetic interactions, namely, magnetic interaction energy within each ring $E_{\text{ex}(\text{rings})}$ and within each chain $E_{\text{ex}(\text{chains})}$ of ellipsoids. Thus, the exchange energy $E_{\text{ex}}$ can be described as

$$E_{\text{ex}} = E_{\text{ex}(\text{rings})} + E_{\text{ex}(\text{chains})} .$$  \hspace{1cm} (S1)

In the following, we shall deduce the mathematical form of the exchange energy $E_{\text{ex}}$ in detail.

The exchange energy $E_{\text{ex}}$ stems from the magnetic interaction between magnetic moments. The energy between two magnetic moments is

$$E_{ij} = \frac{1}{r_{ij}^3} \bigl[ (\mu_i \cdot \mu_j) - \frac{3}{r_{ij}^2} (\mu_i r_{ij} \cdot \mu_j r_{ij}) \bigr],$$  \hspace{1cm} (S2)

where $\mu_i$ and $\mu_j$ are two dipoles of magnetic moments and $r_{ij}$ is vector between $\mu_i$ and $\mu_j$. Thus, both $E_{\text{ex}(\text{rings})}$ and $E_{\text{ex}(\text{chains})}$ can be deduced from the Eq.(S2).

First, starting from the Eq.(S2), the magnetic interaction energy within each ring $E_{\text{ex}(\text{rings})}$ can be calculated as follows:

$$E_{\text{ex}(\text{rings})} = \frac{N_r}{2} \sum_{j=0}^{N_r-1} \sum_{i=1}^{N_e} E_{ij}$$

$$= \frac{N_r}{2} \sum_{j=0}^{N_r-1} \sum_{i=1}^{N_e} \frac{1}{r_{ij}^3} \bigl[ (\mu_i \cdot \mu_j) - \frac{3}{r_{ij}^2} (\mu_i r_{ij} \cdot \mu_j r_{ij}) \bigr],$$  \hspace{1cm} (S3)

$$= \frac{N_r}{2} \sum_{j=0}^{N_r-1} \sum_{i=1}^{N_e} \frac{1}{r_{ij}^3} \bigl[ (\mu_i \cdot \mu_j) - \frac{3}{r_{ij}^2} (\mu_i r_{ij} \cdot \mu_j r_{ij}) \bigr]$$

where each ellipsoid is treated as a dipole of magnetic moment $\mu_e$ and the $N_r$ and $N_e$ represent the number of rings in a nanotube and number of ellipsoids in a ring, respectively. As shown in the Eq.(S3), there are two items of $r_{ij}$ and $\mu_e r_{ij}$ to be solved before the final mathematical form of $E_{\text{ex}(\text{rings})}$ is obtained. To solve $r_{ij}$ and $\mu_e r_{ij}$, we
schematically illustrate the relation between the \( \mu_e(i) \) and \( \mu_e(j) \) in a ring in the Fig. S1 according to the coordinate described in Fig. 1(c). 

(1) Calculation of \( r_{ij} \) item. As shown in Fig. S1, the azimuthal angle of \( \mu_e(i) \) and \( \mu_e(j) \) in ring plane are \( \varphi_i \) and \( \varphi_j \), and there are \( i-1 \) ellipsoids between \( \mu_e(i) \) and \( \mu_e(j) \). Thus, the distance \( r_{ij} \) between \( \mu_e(i) \) and \( \mu_e(j) \) can be calculated as:

\[
 r_{ij} = 2(R - r_e)^2 - 2(R - r_e)^2 \cos(\varphi_i - \varphi_j) = \left( 2(R - r_e)^2 \left[ 1 - \cos(2\pi i / N_e) \right] \right)^{\frac{1}{2}}. \tag{S4}
\]

(2) Calculation of \( \mu_e r_{ij} \) item. According to the mathematical definition, \( \mu_e r_{ij} = \mu_e r_{ij} \cos(<\mu_e, r_{ij}>) \). As shown in Fig. S1, the projection of all magnetic moments in the ring plane (the dotted arrows in the Fig. S1) is parallel to the X-Z plane (the dotted line in the Fig. S1); thus, the angle between projection of \( \mu_e(i) \) and \( r_{ij} \) becomes \( \pi/2 + (\varphi_i + \varphi_j)/2 \). At the same time, the external field \( H \) respectively makes the angles \( (\alpha + \theta_0)/(-\alpha + \theta_0) \) and \( \theta_0 \) with respect to the magnetic moment \( \mu_e \) and tube axis direction according to the coordinate described in Fig.1(c), and we correspondingly show the relation among \( (\alpha + \theta_0)/(-\alpha + \theta_0) \), \( \theta_0 \) and \( H \) in the Figure S2. Thus, the angle between the magnetic moment \( \mu_e \) and the ring plane becomes \( \pi/2 - \alpha \) and the cosine of the angle of \( \mu_e(i) \) to \( r_{ij} \) is calculated as:

\[
 \cos(<\mu_e r_{ij}>) = \cos\left[ \frac{\pi}{2} - \alpha \right] \cos\left[ \frac{\pi}{2} + \varphi_j + \frac{\pi}{N_e} i \right] = - \sin \alpha \sin(\varphi_j + \frac{\pi}{N_e} i). 
\]

Therefore, the \( \mu_e r_{ij} \) can be written as:

\[
 \mu_e r_{ij} = \mu_e r_{ij} \cos(<\mu_e, r_{ij}>) = \mu_e r_{ij} \sin \alpha \sin(\varphi_j + \frac{\pi}{N_e} i). \tag{S5}
\]

Substituting Eq.(S4) and Eq.(S5) for Eq.(S3), the magnetic interaction energy within each ring \( E_{ex(rings)} \) can be written as
\[ E_{\text{ex(rings)}} = \frac{N_r}{2} \sum_{j=0}^{N-1} \sum_{i=1}^{N-1} \frac{1}{r_{ij}^3} \left[ \mu_i^2 - \frac{3}{r_i^2} (\mu_i r_{ij})^2 \right] \]

\[ = \frac{N_r \mu_e^2}{2} \sum_{j=0}^{N-1} \sum_{i=1}^{N-1} \left[ 1 - \frac{3}{N_e} \left( 1 - \cos^2 \alpha \right) \sin^2 \left( \varphi_j + \frac{\pi i}{N_e} \right) \right] \]

(S6)

Second, the magnetic interaction energy within a chain \( E_{\text{ex(chains)}} \) can also be calculated from the Eq.(S2). Considering the exchange energy \( E_{\text{ex(chain)}} \) in a single chain, it is denoted as the sum of all exchange energy between the magnetic moments in a chain, namely, from the nearest-neighbor exchange energy to the furthest-neighbor exchange energy. Here we set the number of rings is even. Thus, the total energy in a single chain can be calculated as:

\[ E_{\text{ex(chain)}} = \sum E_{\text{ex(nearest)}} + \sum E_{\text{ex(next-nearest)}} + \ldots E_{\text{ex(furthest)}} \]

\[ = \frac{1}{r_{ij}^3} \left[ \mu_i^2 - \frac{3}{r_i^2} (\mu_i r_{ij})^2 \right] \sum_{\text{next-nearest}} + \ldots \]

\[ = \frac{(N_r-1) \mu_e^2}{(1 \times 2 k_r)^3} \left( \cos 2\alpha - 3 \cos^2 \alpha \right) + \frac{(N_r-2) \mu_e^2}{(2 \times 2 k_r)^3} \left( 1 - 3 \cos^2 \alpha \right) \]

\[ + \ldots \]

\[ = \frac{N_r K_{N_r} \mu_e^2}{(2k_r)^3} \left[ L_{N_r} \left( \cos 2\alpha - 3 \cos^2 \alpha \right) + M_{N_r} \left( 1 - 3 \cos^2 \alpha \right) \right] \]

where

\[ L_{N_r} = \sum_{i=1}^{\frac{1}{2}(N_r-1)-\frac{1}{2}(N_r+1)} \frac{N_r - (2i - 1)}{N_r(2i - 1)^3} \]

\[ M_{N_r} = \sum_{i=1}^{\frac{1}{2}(N_r-2)-\frac{1}{2}N_r} \frac{N_r - 2i}{N_r(2i)^3} \]

\[ K_{N_r} = M_{N_r} + L_{N_r} = \sum_{i=1}^{N_r} \frac{N_r - i}{N_r i^3} \]

Finally, by substituting Eq.(S6) and Eq.(S7) into Eq.(S1), the total exchange energy \( E_{\text{ex}} \) can be described in the following mathematical form:
\begin{align*}
E_{ex} &= \frac{N_e \mu_e^2}{2} \sum_{j=0}^{N_e} \sum_{i=1}^{N_e-1} \frac{1-3 \left(1-\cos^2 \alpha \right) \sin^2 \left( \phi_j + \frac{\pi i}{N_e} \right)}{2 \left( R-r_i \right)^2 \left[ 1-\cos \left( \frac{2\pi i}{N_e} \right) \right]^3} \right) + \frac{N_e N_e K \mu_e^2}{(2k r)^3} \left[ L_N \left( \cos 2\alpha - 3 \cos^2 \alpha \right) \right] + M_N \left( 1-3 \cos^2 \alpha \right) \right].
\end{align*}

(S8)

Figure. S1

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure_s1}
\caption{Schematic diagram of geometric relation between $\mu_{e(i)}$ and $\mu_{e(j)}$.}
\end{figure}

Figure. S2

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure_s2}
\caption{Angles between magnetic moment $\mu_e$, axis direction of nanotube, and external field $H$.}
\end{figure}
2. Detailed calculation of coercivity in the fanning rotation and coherent rotation

According to the Eq. (11) in our previous work (see the ref. [24] of this article) and the Eq. (9) in this article, we can easily obtain the coercivity $H_c$ of coherent rotation and fanning rotation. The equations are as follows:

For the coherent rotation,

$$H_c = \begin{cases} 
-\frac{2a}{b} \left( \cos^2 \theta_0 + \sin^2 \theta_0 \right)^{\frac{3}{2}}, & 0^\circ < \theta_0 \leq 45^\circ \\
-\frac{as\sin 2\theta_0}{b}, & 45^\circ < \theta_0 \leq 90^\circ 
\end{cases}, \quad (S9)
$$

where

$$a = \frac{N_r \mu_e^2 \sum_{j=0}^{N-1} \sum_{i=1}^{N-1} 3\sin^2 \left( \varphi_j + \frac{\pi i}{N_e} \right)}{2 \left( 2(R - r_e) \right) [1 - \cos \left( 2\pi i / N_e \right)]^{3/2}} - 3 \frac{N_r N_e K_N \mu_e^2}{(2kr_e)^3}$$

$$- \frac{1}{2} N_r N_e I_e \mu_e (N_r - N_o) \, ,$$

$$b = N_r N_e \mu_e \, .$$

For the fanning rotation,

$$H_c = \begin{cases} 
-\frac{2d}{e} \left( \cos^3 \theta_0 + \sin^3 \theta_0 \right)^{\frac{3}{2}}, & 0^\circ < \theta_0 \leq 45^\circ \\
-\frac{d}{e} \sin 2\theta_0, & 45^\circ < \theta_0 \leq 90^\circ 
\end{cases}, \quad (S10)
$$

where

$$d = \frac{N_r \mu_e^2 \sum_{j=0}^{N-1} \sum_{i=1}^{N-1} 3\sin^2 \left( \varphi_j + \frac{\pi i}{N_e} \right)}{2 \left( 2(R - r_e) \right) [1 - \cos \left( 2\pi i / N_e \right)]^{3/2}}$$

$$- \frac{N_r N_e K_N \mu_e^2 L_N}{(2kr_e)^3} - 3 \frac{N_r N_e K_N \mu_e^2 M_N}{(2kr_e)^3}$$

$$+ \frac{1}{2} N_r N_e I_e \mu_e (N_r - N_o) \, ;$$

$$e = \frac{N_r}{2} N_e \mu_e \, .$$
It is clearly seen that there are four coefficients of $a, b, d$ and $e$ in the Eq. (S9) and Eq. (S10); all these coefficients are function of the geometric parameters of nanotubes, including axis ratio $k$, the length of nanotube $N_r$, the number of ellipsoid in a rings $N_e$ and the thickness of nanotube, $2r_e$. To simply calculate the coercivity, we set $N_e$ is equal to 50 and then study the influence of the geometric parameters of nanotube on the magnetic properties, especially for the coercivity $H_c$. All the calculation results are demonstrated in the Fig. 3, Fig. 4 and Fig.5. In addition, in order to compare the coercivity of fanning rotation with that of coherent rotation, we set the same values of $k$, $N_r$, and $2r_e$ to calculate magnetic properties, as shown in the Fig. 5.