Supporting Information

Addition of low concentrations of an ionic liquid to a base oil reduces friction over multiple length scales: a combined nano- and macrotribology investigation

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Calculation of Herzian contact stress

The effective Young’s modulus $E_w$ can be calculated by

$$\frac{1}{E_w} = \frac{1}{2} \left( \frac{1 - v_1^2}{E_1} + \frac{1 - v_2^2}{E_2} \right), \quad (1)$$

where $E_1$, $E_2$ are the elastic moduli and $v_1$, $v_2$ the Poisson’s ratios associated with each body.

Contact area of radius $a$, $\alpha \cong (2RF)/E_w^{1/3}$, \quad (2)

where $R$ is the sphere radius, $F$ is applied load.

Contact stress: $p_{\text{max}} = \frac{3F}{2\pi a^2}$ \quad (3)

The parameters used: Silica: $E=73$ GPa $v=0.17$; silicon: $E=165$ GPa, $v=0.27$; Silicon nitride: $E=231.5$ $v=0.25$

Therefore, in the nanotribology tests

$F=160$ nN, $p_{\text{max}} \approx 1.2 \text{ GPa}$

$F=300$ nN, $p_{\text{max}} \approx 2.1 \text{ GPa}$

Macrotribology test
F=2N, $p_{max} \approx 1.2 \text{ GPa}$

F=10N, $p_{max} \approx 2.1 \text{ GPa}$

**Hamrock and Dowson model**

This model is based on the Hertz model of contact stresses. It is discussed in detail in Stachowiajk’s book *Engineering Tribology*.\(^1\) In this model, the central film thickness, $h_c$, can be calculated in the following equation:

$$h_c = \frac{2.69}{a} \left( \frac{V \eta_0}{E_w a} \right)^{0.67} \left( \frac{a E_w}{F} \right)^{0.53} \left( \frac{a}{E_w} \right)^{-0.067} \left( 1 - 0.61e^{-0.73k} \right)$$

(4)

where $a$ is the contact area radius from eq(2), $V$ is sliding velocity, $\eta_0$ is the viscosity of the lubricant at ambient pressure, $E_w$ is the effective Young’s modulus from eq(1), $\alpha$ is the pressure-viscosity coefficient, $F$ is the applied load, and $k$ is the ellipticity parameter. $k=1$ for point contact. The pressure-viscosity coefficient of hexadecane varies from 11.6 to 13.2, an average of 12.5 is used here.\(^2\) For the pure IL used in this study, the pressure-viscosity coefficient was not found in the literature. According to previous studies, the pressure-viscosity coefficient of ILs are generally between 12~21 GPa\(^{-1}\).\(^3\)-\(^6\) thus here the film thicknesses of the systems are calculated by assuming the limit values of 12 GPa\(^{-1}\) and 21 GPa\(^{-1}\) for the pure IL and the pressure-viscosity coefficients of the mixtures are calculated by molar ratio, as shown in Table S1 and Table S2.

The table below show the calculated film thicknesses do not vary significantly for different chosen values of the pressure-viscosity coefficient, especially for the IL-hexadecane mixtures.

**Table S1.** The pressure-viscosity coefficients and calculated film thickness when assuming the pressure-viscosity coefficient ($\alpha$) of pure IL is 12.

<table>
<thead>
<tr>
<th></th>
<th>hexadecane</th>
<th>0.01 mol% IL</th>
<th>1.0 mol% IL</th>
<th>10 mol% IL</th>
<th>Pure IL</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F=2N$</td>
<td>2.6</td>
<td>2.7</td>
<td>2.8</td>
<td>4.0</td>
<td>118</td>
</tr>
<tr>
<td>$F=10N$</td>
<td>2.4</td>
<td>2.5</td>
<td>2.6</td>
<td>3.5</td>
<td>106</td>
</tr>
</tbody>
</table>

Table S2. The pressure-viscosity coefficients and calculated film thickness when assuming the pressure-viscosity coefficient ($\alpha$) of pure IL is 21.

<table>
<thead>
<tr>
<th></th>
<th>0.01 mol% IL</th>
<th>1.0 mol% IL</th>
<th>10 mol% IL</th>
<th>Pure IL</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>12.5</td>
<td>12.6</td>
<td>13.4</td>
<td>21.0</td>
</tr>
<tr>
<td>Calculated Film thickness (nm)</td>
<td>2 N</td>
<td>2.7</td>
<td>2.8</td>
<td>4.1</td>
</tr>
<tr>
<td></td>
<td>10 N</td>
<td>2.5</td>
<td>2.6</td>
<td>3.7</td>
</tr>
</tbody>
</table>

Table S3. Zero shear viscosity of IL-hexadecane mixtures at 20 °C measured by an AR-G2 rheometer (TA instruments).

<table>
<thead>
<tr>
<th>IL concentration</th>
<th>hexadecane</th>
<th>0.01 mol%</th>
<th>1 mol%</th>
<th>10 mol%</th>
<th>Pure IL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Viscosity (mPa s)</td>
<td>3.8</td>
<td>3.9</td>
<td>4.0</td>
<td>6.7</td>
<td>1007</td>
</tr>
<tr>
<td>Density (g/cm$^3$)</td>
<td>0.773$^7$</td>
<td>0.773$^a$</td>
<td>0.776$^a$</td>
<td>0.802$^a$</td>
<td>0.885$^8$</td>
</tr>
</tbody>
</table>

$^a$The values are calculated from the weight concentration of the IL and the hexadecane in the mixture.

7. M. S. D. S. Hexadecane, Sigma-Aldrich.