Supporting Information

Steering Microtubule Shuttle Transport with Dynamically Controlled Magnetic Fields


Supplementary Movies and Figures

Supplementary Figure 1: Geometry used in analysis of Supplementary Movies. QI, QII, QIII, and QIV define the standard Cartesian coordinate naming convention of (+, +), (-, +), (-, -), and (+, -) values of (x, y), respectively. Each (x, y) value was converted into polar coordinates consisting of a vector \( \vec{r} \), representing the distance from the vertex to the MT leading tip, and \( \theta \), the angle of \( \vec{r} \) with respect to the x axis as shown. In addition, the angle \( \phi \), the angle with respect to the vertex for entry and exit, was calculated. The angle between the vertex arm and the x-axis is 51°. Thus, an exit angle perpendicular to the x-axis and the vertex tip is 39°, and an exit angle parallel to the second zig-zag arm would be 78°. For Movie S1, the origin was the lower left corner of the image, thus all values were in QI, and \( \theta \) was calculated using the Cartesian geometry as shown.
Supplementary Figure 2: Length Distribution of MTs investigated in Movie S1. The average length for the 100 MTs investigated 7.3 ± 3.3 µm.

Supplementary Movie Captions

Movie S1: Microtubule shuttles bound to fluorescent MagDots (modification ratio ~ 23 MagDots/µm MT length) can be induced to move on kinesin-coated surface via ATP-directed processes in the absence of magnetic fields. Scale bar = 4 µm.

Movie S2: A microtubule shuttle changes its direction of motion as a result of the magnetic force of two nearby vertices. The out-of-plane magnetic field, initially present, is reversed at 8 seconds, explaining why the microtubule is unaffected by subsequent wire vertices. Modification ratio is 20 MagDots/µm. Scale bar = 4 µm.

Movie S3: A microtubule shuttle changes its direction of motion as a result of the magnetic force of a nearby vertex. The out-of-plane magnetic field, initially present, is reversed at 5 seconds, explaining why the microtubule is unaffected by subsequent wire vertices. Modification ratio is 13 MagDots/µm. Scale bar = 4 µm.
Movie S4: A microtubule shuttle forms an open circular shape and moves in circular motion as a result of transient tip-pinning by magnetic trapping at a nearby vertex. Magnetic trapping is active throughout the viewing period. Modification ratio is 23 MagDots/µm. Scale bar = 4 µm.

**Calculation of Microtubule Bending Strain Energy**

Using the following general equation derived for a rod:

\[
\frac{dU}{L} = \int_0^L \left( \frac{f^2}{2E} \right) \cdot (dS)(dA)
\]

(Equation 1)

In this equation, L is the contour length of the rod, f is the bending stress acting on a cross-section δA, E is the Young’s modulus of the rod, and (δS)(δA) is a volume element. Taking into account that the bending stress, f is proportional to the torque (bending moment = M) and inversely proportional to the moment of inertia of the cross-section (I), and that the torque (M) is inversely related to the radius of curvature (R, assumed to be a constant) of the rod by the equation,

\[ M = \frac{EI}{R} \]

(Equation 2)

the general energy equation (Equation 1) can be simplified as follows:

\[ U = \frac{EI L}{2R^2} \]

(Equation 3)

where I is the moment of inertia of the cross-section, R is the radius of curvature of the rod (assumed to be a constant). The product EI (called flexural rigidity) is proportional to the persistence length (Lp) of the rod according to the following relation:

\[ EI = L_p \cdot k_B T \]

(Equation 4)

in which k_B and T are respectively the Boltzmann constant and the temperature. Therefore, the energy required to bend the microtubules can be written as follows:
\[ E_{\text{bend}} = \frac{k_B T L_p L}{2R^2} \]  

(Equation 5)

Assuming a persistence length, \( L_p \), of 3000 \( \mu \text{m} \) and replacing the contour length \( L \) as well as the average radius of curvature by their actual values (Table 2), the strain energy required to bend each microtubule was measured.

**Calculation of Strain Energy Induced by Kinesin Motors**

The strain induced by kinesin alone is calculated by multiplying the work done by each kinesin motor with the number of motors bound to the MTs. The energy or work per kinesin motor is calculated by taking the motor force of \(~5 \text{ pN}\) over the motor step distance of 8 nm giving \( 4 \times 10^{-20} \text{ J} \), which is \(~10 \text{ kT} \). The number of motors bound to the MTs is calculated by comparing the surface density of kinesin to the projected area of MTs. For example, for a MT 3 \( \mu \text{m} \) in length and 25 nm in diameter, the projected area is 0.075 \( \mu \text{m}^2 \). Comparing this area with estimated kinesin surface densities of 7,800 \( \mu \text{m}^{-2} \), estimated based on a previously reported method,\(^3\) yields \(~590 \) motors bound to a 3 \( \mu \text{m} \) long MT. Multiplying the number of motors bound by the work energy of a single motor yields a kinesin-induced strain estimate in the absence of a magnetic field.

**REFERENCES**