

### Supplementary information

- We expect the continuous desorption to occur even for an infinite filament. The time series of the bound fraction of motor proteins,  $\psi_b = N_b/N_b^t$ , at different transverse pulling forces are shown in Fig. 1. Here  $N_b^t$  is the maximum number of motors that may attach to the filament, given the motor-density on the bed, filament length and capture radius of each motor protein. This shows that there is no coexistence of adsorbed and partially desorbed states, a hallmark of first order phase transitions.

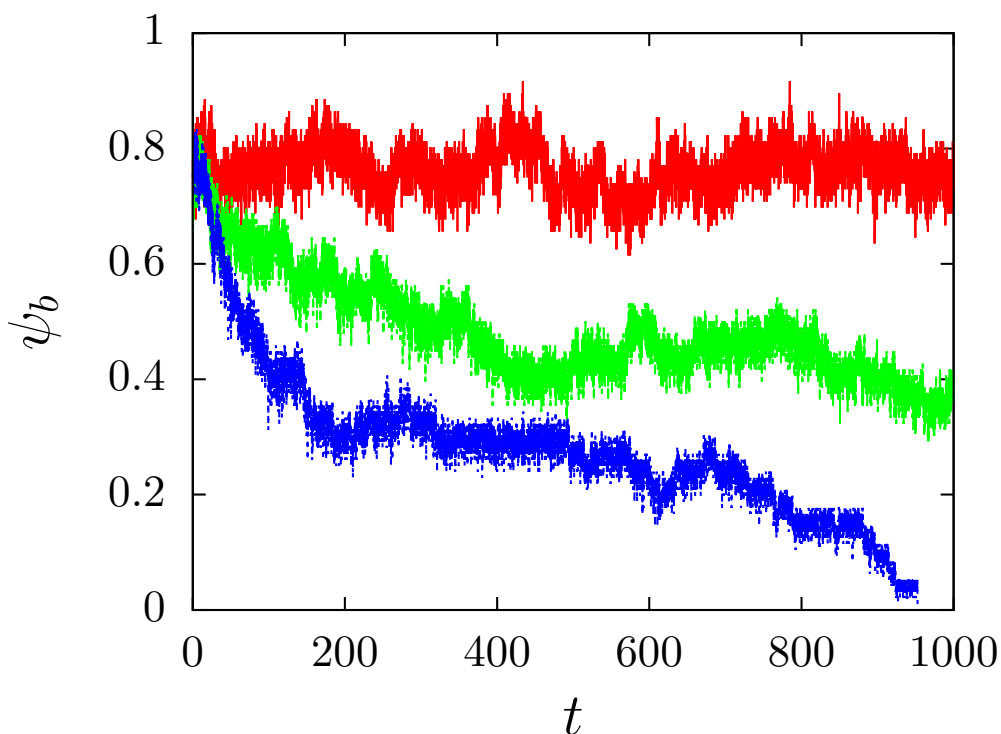


Figure 1: Time series of the bound fraction of motor proteins at transverse pulling forces  $F_z = 16$  (red), 20 (green) and at the critical desorption force  $F_z = 22$  pN.

The total amount of desorbed length of the polymer is independent of the total contour length, and depends only on the applied force. The amount of critical desorption force  $F_z^c$  is proportional to the polymer contour length  $L_c$ , for a given motor-protein density. This can be seen from Eq. 5 (*main text*), by noting that  $N_b \propto L_c$ . In the thermodynamic limit of large  $L_c$ , we expect  $F_z^c/L_c$  to attain a constant value.

- The probability of spontaneous desorption of a filament of finite length, as considered in our paper, is extremely small but finite. If all attached motor proteins detach simultaneously, this could happen. Note that if the polymer

had only a single monomer, spontaneous desorption probability is  $1 - \Omega_d$ , in absence of external force. For the value of  $\Omega_d = 20/21$  considered in our paper, this is equal to 0.05. For a polymer having  $N$  monomers, the probability goes down to  $(1 - \Omega_d)^N$ . For example, for a polymer having 30 monomers this gives us a spontaneous desorption probability  $2 \times 10^{-40}$ .

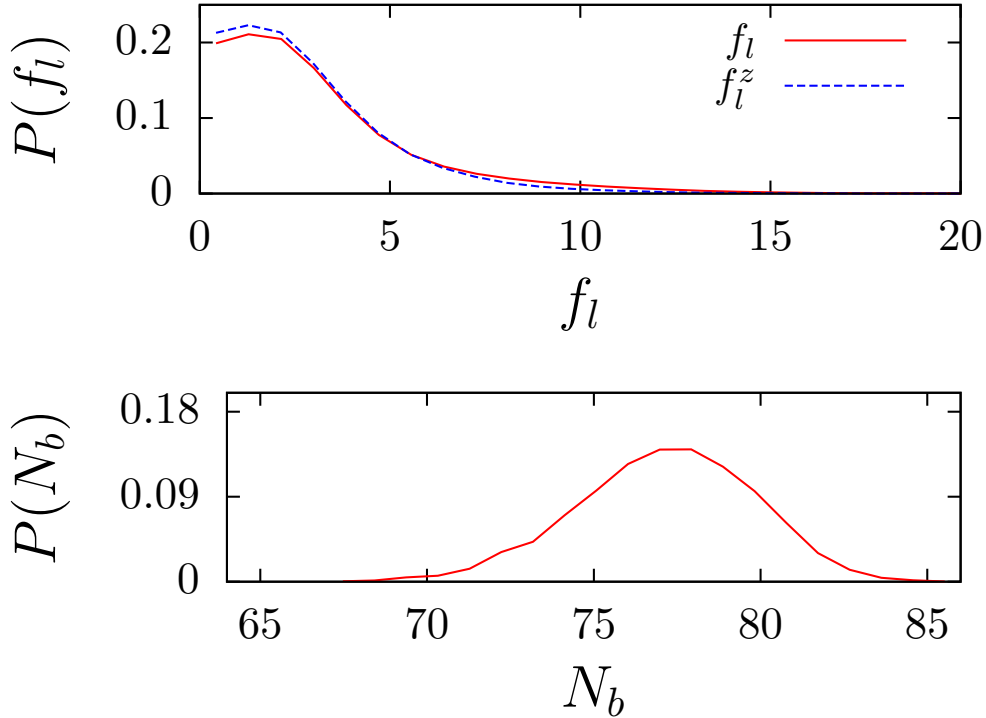


Figure 2: (Top) Probability distribution of the load force  $f_l = k_m |\delta \mathbf{r}|$  (solid line, red) on individual bound motors due to pulling  $F_z = 17$  on microtubule, shows a peak at  $f_l \approx 1.3$ . In mean field theory, only the  $z$ -component of load is considered, and the distribution of corresponding values obtain from simulations is presented using dotted line (blue). This shows a peak near 1.21. Compare this to the value  $F_z / \langle N_b \rangle \approx 0.22$ , the mean field estimate given the numerical estimate  $\langle N_b \rangle \approx 77$ . (Bottom) Probability distribution of bound motors  $N_b$ .

- The force distribution on individual bound motors as obtained from the simulation for a partially desorbed polymer (Fig. 2(top)) is shown. This distribution in the presence of  $F_z$  shows a peak with a somewhat broad distribution. The distribution is related to the distribution of bound motors, presented in Fig. 2(bottom). The peak of mean value obtained from  $P(f_l)$ , however, is not equal to  $F_z / \langle N_b \rangle$ . One reason is stochastic forces neglected in the mean-field analysis. Equilibrium thermodynamics predicts :  $k_m \langle (\delta z)^2 \rangle = k_B T$ , where  $k_m$  is the spring constant of the motor,  $\delta z$  corresponds to a displacement in the  $z$  direction and  $T$  is the temperature. Assuming that room temperature is

the only source of stochasticity, this gives the thermal force on motor-proteins  $k_m \sqrt{\langle \delta z^2 \rangle} \sim 1$  pN for the parameter values used in the simulations. Subtracting this value from the peak value of  $P(f_l)$ , one obtains values close to the mean field expectation  $F_z / \langle N_b \rangle$ . Note that attachment detachment kinematics acts as another source of stochasticity in the problem.

Another point to note, using  $f_l = F_z / \langle N_b \rangle$ , strictly, one assumes a delta-function distribution of load, which is not the case in reality (Fig. 2(top)).