Supplementary information

Title: Size-Dependent Hardness of Five-fold Twin Structured Ag Nanowires

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Double Contact Model

The details of this model can be found in the work by Feng et al.[22, 29] and only the key ideas will be presented here to help understanding of the results. To apply the model, the required inputs are indentation load $P$, indentation stiffness $S$, tip radius $R_1 (=112\text{nm})$, nanowire radius $R_2$, as well as reduced moduli for contacts 1 and 2, i.e., $E_{r1}$ and $E_{r2}$, where $E_{r1}$ and $E_{r2}$ can be determined by the moduli of substrate ($E_s$), indenter ($E_i$) and nanowire ($E_n$). The following are the key set of analytical equations for solving the nanowire indentation hardness $H_n$.

\[
H_n = H_1 = \frac{P}{A_{k_1}} = \frac{P}{\pi k_1 a_1^2} \quad k_1 = \frac{b_1}{a_1} = \left(1 + \frac{R_1}{R_2}\right)^{-2/3} \tag{S1}
\]

\[
\frac{1}{S} = \frac{1}{S_1} + \frac{1}{S_2}, \quad S_1 = \lambda_1 2E_{r1}a_1\sqrt{k_1}, \quad S_2 = \lambda_2 2E_{r2}\sqrt{a_2b_2} \tag{S2}
\]

\[
\lambda_1 = \frac{\pi}{2\sqrt{k_1 K_1'}}, \quad K_1' = \int_0^{\pi} \frac{1}{\sqrt{1 - (1 - k_1^2)\sin^2 \theta}} d\theta \tag{S3}
\]

\[
\lambda_2 = \frac{\pi}{2\sqrt{k_2 K_2'}}, \quad K_2' = \int_0^{\pi} \frac{1}{\sqrt{1 - (1 - k_2^2)\sin^2 \theta}} d\theta \quad k_2 = \frac{b_2}{a_2} \tag{S4}
\]

\[
H_2 = \frac{P}{A_{k_2}} = \frac{P}{\pi a_2b_2}, \quad b_2 = \sqrt{\frac{2PR_2}{\pi a_2E_{r2}}}, \quad a_2 = 2R_2 \left[\frac{2.2 - \alpha}{1+\alpha}\right]^{0.342} + \frac{s}{m^{2-m-1}} \left(\frac{a_1}{2R_2}\right)^m \tag{S5}
\]
\[\alpha = \frac{E_s - E_{sp}}{E_s + E_{sp}},\ E_s = \frac{E_n}{1 - v_n^2},\ E_{sp} = \frac{E_n}{1 - v_s^2}\]  

(S6)

\[s = 0.8756(\alpha + 0.89)^{0.2311} - 0.11\alpha,\ m = 1.672 - 0.45\alpha + 0.066\alpha^2 + 0.111\alpha^3\]  

(S7)

\[h_{e1} = 3P/2S_1,\ h_{e2} = 3P/2S_2\]  

(S8)

where \(S_1\) and \(S_2\) are the contact stiffnesses for contacts 1 and 2, respectively; \(h_{e1}\) is the elastic displacement of the indenter for contact 1. \(E_{r1}\) and \(E_{r2}\) are the reduced moduli for contacts 1 and 2, respectively, where the reduced modulus is defined in Equation S2. \(a_1\) and \(b_1\) are major and minor radii of contact 1 (shown in Figure 3); \(a_2\) and \(b_2\) are major and minor radii of contact 2 (shown in Figure 3). \(R_1\) is the tip radius, and \(R_2\) is the nanowire radius.

As aforementioned, the nanowire’s hardness \(H_n\) can be determined by \(H_1\) at contact 1, and based on Equation S1, \(k_1\) and \(a_1\) need to be solved to calculate \(H_1\). Here, \(k_1\) can be easily solved by the known indenter radius \(R_1\) and nanowire radius \(R_2\). The solving of \(a_1\) needs to jointly solve Equation S2-S7, which is quite involved as discussed below. First, based on Equation S3, \(\lambda_1\) can be solved only based on \(R_1\) and \(R_2\), and thus, based on Equation S2, the contact stiffness for contact 1, i.e., \(S_1\), becomes a function only of \(a_1\). Then, based on Equation S5 and the required inputs, the contact radii \((a_2\) and \(b_2\)) of elliptical contact 2 are function only of \(a_1\), and thus based on Equations S2 and S4, the contact stiffness of contact 2, i.e., \(S_2\), becomes a function only of \(a_1\) in Equation S2. Consequently, both \(S_1\) and \(S_2\) are functions only of \(a_1\), and then the stiffness equation \((1/S = 1/S_1 + 1/S_2)\) in Equation S2 and experimentally measured stiffness total stiffness \(S\) can be used to solve \(a_1\). Here, \(S\) is fitted with a power law against the measured maximum load at the point of unload \((S = AP^n)\) for each partial unloading. Finally, the nanowire hardness \(H_n\), i.e., the contact pressure in contact 1, can be determined by the solved \(a_1\) and measure load \(P\) by Equation S1.
Reference
