# Electronic Supplementary Information

# Thermomechanical analysis of picograms of polymers using suspended microchannel cantilever

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## Theory

The resonance frequency of the suspended microchannel cantilever is governed by the following equation.

$$f_c(T)|_{T=T_0} = \frac{1}{2\pi}\sqrt{\frac{k_c}{m_c}} \qquad (S1)$$

$$k_c = \frac{E_c w t^3}{4L^3} \qquad (S2)$$

Where $f_c(T)|_{T=T_0}$ is the frequency of the cantilever at temperature $T_0$ (=25 ºC), $k_c$ and $m_c$ are stiffness and mass, respectively, at same temperature. $E_c$, $w$, $t$ and $L$ are the elastic modulus, width, thickness and length of the cantilever, respectively.

Normally, $E_c$ is a strong function of temperature and it varies with temperature in a more complex way than the geometric factors of the cantilever itself. Here we assume that the cantilever material (i.e. silicon nitride) is linearly elastic therefore, the variation of $E_c$ can be expressed:

$$E_c(T) = a(T)E_c \qquad (S3)$$

where $a(T)$ is the coefficient of elastic modulus change with temperature. On the other hand, the geometric factors would change with temperature due to thermal expansion. If the coefficient of thermal expansion (CTE) of silicon nitride is $\alpha$, then with increasing temperature all the dimensional parameters will be the following

$$w(T) = w(1+\alpha T),\ t(T) = t(1+\alpha T),\ L(T) = L(1+\alpha T) \qquad (S4)$$

By letting $b = 1+\alpha T$, eq S2 becomes the following at any temperature T.

$$k_c(T) = \frac{\{Ea(T)\}\{bw\}\{bt\}^3}{\{bL\}^3} = a(T)b\frac{Ewt^3}{L^3} = [a(T)b]k_c \qquad (S5)$$

Using Equation ($S1$), the resonance frequency of the cantilever at any temperature T would be the following

$$f_c(T) = \left[a(T)b\right]^{\frac{1}{2}} f_c(T)|_{T=T_0} \qquad (S6)$$

Changes in $b$ are typically in the ppm range, therefore the $a(T)$ parameter dominates Equation (S6) causing $f_c(T)$ to reduce with increasing temperature. However, in this case, the frequency of the cantilever increases slightly with temperature (<0.01%), potentially due to a release of residual stress in the cantilever.

When a polymer is loaded in the channel, and modelling the system using a parallel spring model (Figure S1), Equation (*S*1) transforms into eq. S7, as given below,

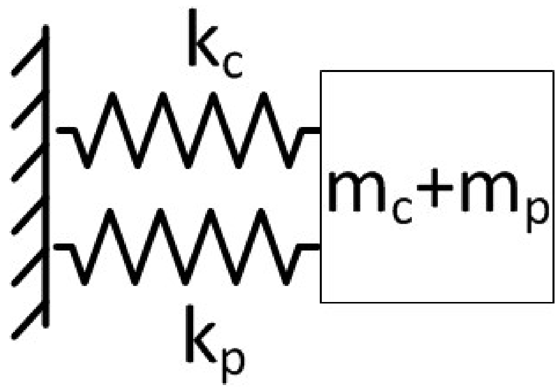


Figure S1. Parallel spring model to describe the suspended microchannel cantilever filled with polymer.

$$f_{c+p}(T)\,|_{T=T_0} = \frac{1}{2\pi}\sqrt{\frac{k_c + k_p}{m_c + m_p}} \qquad (S7)$$

where $k_p$ and $m_p$ are the stiffness and mass of the polymer, respectively.

Equation (*S*2) can be expressed as:

$$f_{c+p}(T)\,|_{T=T_0} = \frac{1}{2\pi}\sqrt{\frac{k_c\left(1+\frac{k_p}{k_c}\right)}{m_c\left(1+\frac{m_p}{m_c}\right)}} \qquad (S8)$$

and thus $$f_{c+p}(T)|_{T=T_0} = \frac{1}{2\pi}\sqrt{\frac{k_c}{m_c}} \left(1+\frac{k_p}{k_c}\right)^{\frac{1}{2}} \left(1+\frac{m_p}{m_c}\right)^{-\frac{1}{2}} \quad (S9)$$

$$f_{c+p}(T)|_{T=T_0} = f_c(T)|_{T=T_0} \left(1+\frac{k_p}{k_c}\right)^{\frac{1}{2}} \left(1+\frac{m_p}{m_c}\right)^{-\frac{1}{2}} \quad (S10)$$

With the function of temperature the effective resonance frequency becomes

$$f_{c+p}(T) = f_c(T)|_{T=T_0} \left(1+\frac{k_p(T)}{k_c}\right)^{\frac{1}{2}} \left(1+\frac{m_p}{m_c}\right)^{-\frac{1}{2}} \quad (S11)$$

Where *$k_p$ (T)* is stiffness of the polymer with function of temperature. From the nature of materials (polymer sample and silicon nitride), it is evident that temperature has a larger influence on $k_p$ than $k_c$. Since the variation of $f_c(T)$ is not large (<0.01%) therefore assuming $f_c(T)$ is constant with increasing temperature which converges to $f_c(T)|_{T=T_0}$ and $C = \left(1+\frac{m_p}{m_c}\right)^{-\frac{1}{2}}$ which is constant within the temperature range of interest due to no mass loss of the polymer we can evaluate $k_p(T)$ from the following Equation ($S$12).

$$k_p(T) = k_c\left[\left(\frac{f_{c+p}(T)}{f_c(T)|_{T=T_0}}\right)^2 C^2 - 1\right] \quad (S12)$$

Equation (S12) has been used to calculate the stiffness of PLA and PMMA. The results are presented in Figure 4 in the main article.

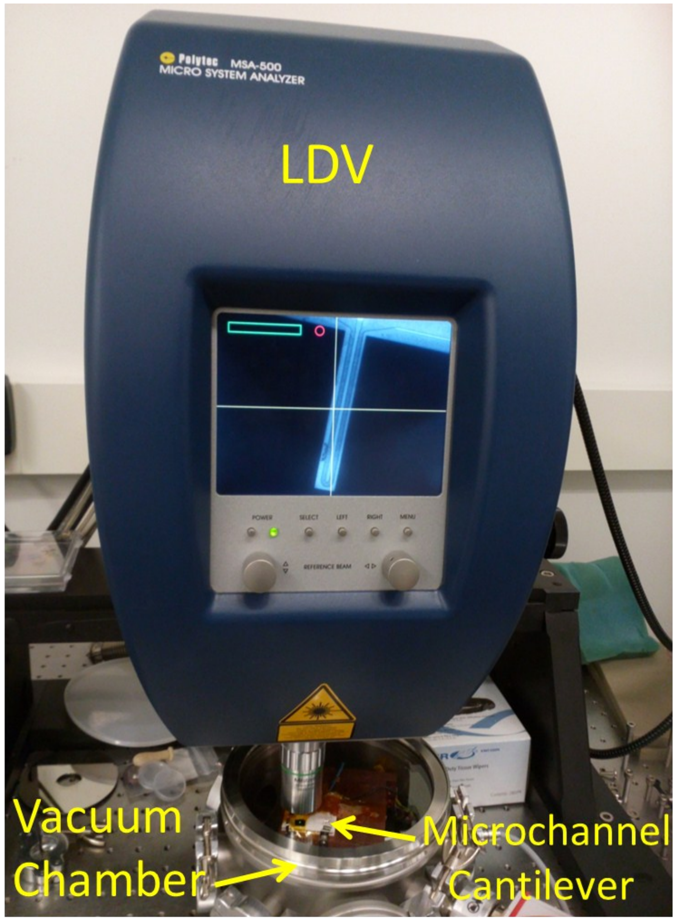


Figure S2: Photograph of the experimental set-up.