Carbon Nanotubes Kirigami Mechanical Metamaterials

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(1) Poisson’s ratios of all the CNT-k

Figure S1 Influence of ratio of surface area of CNT-k to that of pristine CNT on Poisson’s ratio (xz and yz) for the three types of CNT-k with \(\theta\) ranging from 15\(^\circ\) - 75\(^\circ\). (a) and (d) for OCK; (b) and (e) for RCK; (c) and (f) for ECK, respectively.

In the main text, a newly dimensionless geometry parameter \(\alpha\) is defined to characterizes the...
mechanical properties of the CNT-k. In order to calculate $\alpha$ (the ratio of surface area of CNT-k to
that of pristine CNT), unique unit cells for OCK, RCK and ECK are drawn as follow Figures S1-3.

(2) Geometrical Parameter $\alpha$ for OCK

According to the definition of $\alpha$, a vacuum area in the dash orthogon is required to determined. It is
noted that $S_M$ and $S_T$ are the area of kirigami (marked by blue) and orthogonal unit cell (dashed
box).

![Figure S1 Schematic of one orthogonal unit cell where various structural parameters are indicated.](image1.png)

$S_M$ and $S_T$ are calculated as follow;

$$S_T = (2b - L)(4a - 2L) = 8ab - 4aL - 4bL + 2L^2$$

$$S_M = 2a(2b - L) - (2a - 2L)(2b - 2L) + L(4a - 2L - 2a) = 4aL + 4bL - 6L^2$$

The geometrical parameter $\alpha$ for OCK can be therefore obtained by

$$\alpha_{OCK} = \frac{S_M}{S_T} = \frac{4ab + 4bL - 6L^2}{8ab - 4aL - 4bL + 2L^2} = \frac{4bL(1 + \tan \theta) - 6L^2}{8b^2 \tan \theta - 4bL(1 + \tan \theta) + 2L^2}$$

(3) Geometrical Parameter $\alpha$ for RCK

![Figure S2 Schematic of one planar unit cell of RCK where various structural parameters are indicated.](image2.png)
Based on the symmetrical feature of the rhomboid, it can be determined that $KO=OS=QM=MI$, $RN=NJ=LP=PT$ and $\tan \theta = \frac{b}{a}$.

where

$$IK = 2\left(b - \frac{L}{\sin \theta}\right)$$

$$VE = a - \frac{L}{2 \cos \theta}$$

$$DV = b - \frac{L}{\sin \theta}$$

And

$$S_T = XW \cdot WV = 4ab$$

$$S_M = S_T - S_{IKL} - 4S_{EVD}$$

$$S_{IKL} = IK \cdot JL = IK^2 \cdot \tan \theta = \left(b - \frac{L}{\sin \theta}\right)^2 \tan \theta$$

$$S_{EVD} = \frac{VE \cdot DV}{2} = \frac{(a - \frac{L}{2 \cos \theta})(b - \frac{L}{\sin \theta})}{2}$$

Finally, the geometrical parameter $\alpha$ for RCK can be expressed by

$$\alpha_{RCK} = \frac{S_M}{S_T} = \frac{4ab - 2(a - \frac{L}{2 \cos \theta})(b - \frac{L}{\sin \theta}) + (b - \frac{L}{\sin \theta})^2 \tan \theta}{4ab}$$

$$= \frac{4ab \cos \theta \sin \theta}{2} - \frac{2ab \cos \theta}{2} - \frac{2bL \sin \theta}{2} + \frac{L^2}{2} + \frac{2b^2 \sin^2 \theta}{2} - \frac{4bL \sin \theta}{2} + \frac{2L^2}{8ab \sin \theta \cos \theta}$$

(4) Geometrical Parameter $\alpha$ for ECK

Based on the symmetrical feature and the schematic, it can be determined that $KO=OS=QM=MI=RN=NJ=LP=PT=L/2$. 
Figure S3 Schematic of one planar unit cell of ECK where various structural parameters are indicated.

And

\[ ND = \frac{2b + L}{2(2a + L)} \sqrt{4aL + L^2} \]  

(12)

\[ OE = \frac{2a + L}{2(2b + L)} \sqrt{4bL + L^2} \]  

(13)

\[ \theta_{ETR} = \arccos\left(\frac{1}{2b + L} \sqrt{4bL + L^2}\right) \]  

(14)

\[ \theta_{DTR} = \arccos\left(\frac{2a}{2a + L}\right) \]  

(15)

\[ S_r = 4ab \]  

(16)

\[ S_M = 4\left(S_{NDT} + S_{DET} + S_{EOT} - S_{JKT}\right) \]  

(17)

\[ S_{NDT} = \frac{NT \cdot ND}{2} = \frac{a(2b + L)}{4(2a + L)} \sqrt{4aL + L^2} \]  

(18)

\[ S_{EOT} = \frac{OT \cdot OE}{2} = \frac{b(2a + L)}{4(2b + L)} \sqrt{4bL + L^2} \]  

(19)

\[ S_{DET} = \frac{TS \cdot TR}{2} (\theta_{ETR} - \theta_{DTR}) = \frac{(2a + L)(2b + L)}{8} \left(\arccos\left(\frac{1}{2b + L} \sqrt{4bL + L^2}\right) - \arccos\left(\frac{2a}{2a + L}\right)\right) \]  

(20)

\[ S_{JKT} = \frac{\pi \cdot TK \cdot TJ}{4} = \frac{\pi(2a - L)(2b - L)}{16} \]  

(21)

Finally, the geometrical parameter \( \alpha \) for ECK is calculated as

\[ \alpha_{ECK} = \frac{S_{\alpha}}{S_r} = \frac{2aL + aL + 2bL + 2aL + bL}{4ab} \left(\arccos\left(\frac{1}{2b + L} \sqrt{4bL + L^2}\right) - \arccos\left(\frac{2a}{2a + L}\right)\right) - \frac{\pi(2a - L)(2b - L)}{4} \]  

(22)