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Derivation of Eq. (13)

$\frac{\Delta\omega_{\mathrm{cavR}}}{\Delta\omega_{\mathrm{cavR}}+\Delta\omega_{\mathrm{cavNR}}}F$ in Eq. (12) indicates that the radiative component in the Purcell factor can contribute to EM enhancement. In the case of emission, $\frac{\Delta\omega_{\mathrm{cavR}}}{\Delta\omega_{\mathrm{cavR}}+\Delta\omega_{\mathrm{cavNR}}}F$ can enhance the radiative decay rate of spectral component of ω_{em} as

$$k_{\mathrm{r}}(\omega_{\mathrm{em}})+k_{\mathrm{r}}(\omega_{\mathrm{em}})F_1(\omega_{\mathrm{em}})\frac{\Delta\omega_{\mathrm{cavR}}}{\Delta\omega_{\mathrm{cavR}}+\Delta\omega_{\mathrm{cavNR}}}. \quad \text{(S1)}$$

The enhanced total decay rate including higher order Purcell factors $\sum_{l=1}^{\infty}F_l(\omega_{\mathrm{em}})$ (F = F_1, the Purcell factor of dipole mode) is

$$K_{\mathrm{r}}+K_{\mathrm{nr}}+\int_0^{\infty}k_{\mathrm{r}}(\omega_{\mathrm{em}})\sum_{l=1}^{\infty}F_l(\omega_{\mathrm{em}})\mathrm{d}\omega_{\mathrm{em}}, \quad \text{(S2)}$$

where K_{r} ($=\int_0^{\infty}k_{\mathrm{r}}(\omega_{\mathrm{em}})\mathrm{d}\omega_{\mathrm{em}}$) and K_{nr} ($=\int_0^{\infty}k_{\mathrm{nr}}(\omega_{\mathrm{em}})\mathrm{d}\omega_{\mathrm{em}}$) are radiative and non-radiative decay rate of a molecule in a free space, respectively, where $k_{\mathrm{r}}(\omega_{\mathrm{em}})\mathrm{d}\omega_{\mathrm{em}}$ and $k_{\mathrm{nr}}(\omega_{\mathrm{em}})\mathrm{d}\omega_{\mathrm{em}}$ are radiative and non-radiative decay rate at ω_{em}, respectively. Thus, a radiative efficiency of molecule expressed as

$$\frac{k_{\mathrm{r}}(\omega_{\mathrm{em}})}{K_{\mathrm{r}}+K_{\mathrm{nr}}} \quad \text{(S3)}$$

is changed into

$$\frac{k_{\mathrm{r}}(\omega_{\mathrm{em}})+k_{\mathrm{r}}(\omega_{\mathrm{em}})F_1(\omega_{\mathrm{em}})\frac{\Delta\omega_{\mathrm{cavR}}}{\Delta\omega_{\mathrm{cavR}}+\Delta\omega_{\mathrm{cavNR}}}}{K_{\mathrm{r}}+K_{\mathrm{nr}}+\int_0^{\infty}k_{\mathrm{r}}(\omega_{\mathrm{em}})\sum_{l=1}^{\infty}F_l(\omega_{\mathrm{em}})\mathrm{d}\omega_{\mathrm{em}}}. \quad \text{(S4)}$$

Thus, the enhancement factor of emission intensity is described as

$$\frac{\dfrac{k_{\mathrm{r}}(\omega_{\mathrm{em}})+k_{\mathrm{r}}(\omega_{\mathrm{em}})F_1(\omega_{\mathrm{em}})\dfrac{\Delta\omega_{\mathrm{cavR}}}{\Delta\omega_{\mathrm{cavR}}+\Delta\omega_{\mathrm{cavNR}}}}{K_{\mathrm{r}}+K_{\mathrm{nr}}+\int_0^{\infty}k_{\mathrm{r}}(\omega_{\mathrm{em}})\sum_{l=1}^{\infty}F_l(\omega_{\mathrm{em}})\mathrm{d}\omega_{\mathrm{em}}}}{\dfrac{k_{\mathrm{r}}(\omega_{\mathrm{em}})}{K_{\mathrm{r}}+K_{\mathrm{nr}}}}=\frac{1+F_1(\omega_{\mathrm{em}})\dfrac{\Delta\omega_{\mathrm{cavR}}}{\Delta\omega_{\mathrm{cavR}}+\Delta\omega_{\mathrm{cavNR}}}}{1+\dfrac{\int_0^{\infty}k_{\mathrm{r}}(\omega_{\mathrm{em}})\sum_{l=1}^{\infty}F_l(\omega_{\mathrm{em}})\mathrm{d}\omega_{\mathrm{em}}}{K_{\mathrm{r}}+K_{\mathrm{nr}}}}. \quad \text{(S5)}$$

In Eq. (S5), if one makes approximation as

$$\int_0^{\infty}k_{\mathrm{r}}(\omega_{\mathrm{em}})\sum_{l=1}^{\infty}F_l(\omega_{\mathrm{em}})\mathrm{d}\omega_{\mathrm{em}}\approx K_{\mathrm{r}}\sum_{l=1}^{\infty}F_l(\omega_{\mathrm{em}}). \quad \text{(S6)}$$

One can derive the denominator of Eq. (13).