Electronic Supplementary Information for

Rocket-inspired tubular catalytic microjet with grating-structured wall as guiding empennages

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1. The force distribution acting on a groove of the microjet.

Figure S1 presents the coordinate system used in the calculation. Here, we choose the frame of reference attached to the microjet. Thus, the grating-structured microjet is relatively static and the flow is moving toward the microjet. The x-direction is parallel to the grating structure (i.e., groove). θ is defined as the angle between x axis and the flow velocity vector.

Due to the existence of grating structure on the tube wall, the fluid that flows through the groove forms streamtube is constrained by the groove.[S1,S2] That is, the fluid flows only along the streamtube.

To investigate the motion in detail, an arbitrary infinitesimal element of streamtube with control volume is studied.[S3] The upper panel of Figure S1 shows that the fluid flows through the control volume from the inlet section A_1 and outflows from section A_2. In steady case, the flow has a uniform inlet flow (p_1, A_1, U_1) and a uniform exit flow (p_2, A_2, U_2).

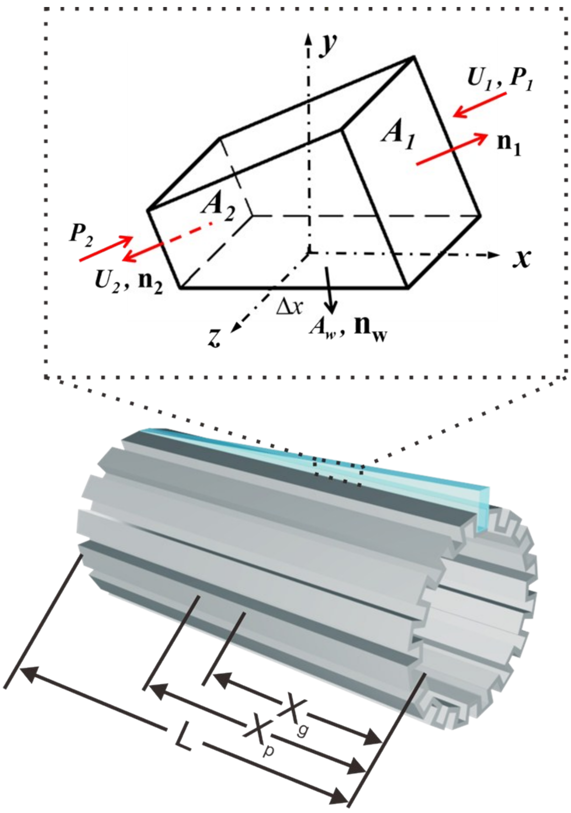


Figure S1. Coordinate system used in the theoretical model.

The momentum of the element with control volume denoted by mass dm can be expressed as:

$$\mathbf{U}dm = \rho \mathbf{U} dV \quad \text{(s1)}$$

where ρ is the density of the fluid. In Newton's second law, the sum of all forces acting on the control volume is equal to the change rate of momentum:

$$\frac{D}{Dt}\iiint_V \rho \mathbf{U} dV = \sum \mathbf{F}, \tag{s2}$$

where V is the volume of the control volume. According to Reynolds transport theorem,[S4] the term on the left of equation (s2) is the change rate of the momentum that includes change rate of momentum inside the control volume and the net outward momentum flux across the boundary:

$$\frac{D}{Dt}\iiint_V \rho \mathbf{U} dV = \iiint_V \frac{\partial(\rho \mathbf{U})}{\partial t} dV + \oiint_S \rho \mathbf{U}(\mathbf{U}\cdot\mathbf{n}) dA. \tag{s3}$$

The term $\Sigma\mathbf{F}$ is the vector sum of all forces acting on the control volume, which includes the body force acting on the fluid inside the control volume and the surface forces acting on the boundary. Let **f** represent the net body force per unit mass exerted on the fluid inside the control volume.[S5] The body force on the elemental volume dV is $\rho \mathbf{f} dV$ and the total body force exerted on the fluid in the control volume is the summation of the above over the volume V. The elemental surface force due to pressure acting on the element of area dA is $-P\mathbf{n}dA$ and the negative sign indicates that the force is in the direction opposite of dA. Thus,

$$\sum \mathbf{F} = \iiint_V \rho \mathbf{f} dV - \oiint_S P\mathbf{n} dA. \tag{s4}$$

Since the fluid is considered to be incompressible, the time-derivative volume integral is zero:

$$\frac{\partial}{\partial t}\iiint_V \rho \mathbf{U} dV = 0. \tag{s5}$$

In addition, the microjet moves in the horizontal plane of flow field and the body force is zero:

$$\iiint_V \rho \mathbf{f} dV = 0. \tag{s6}$$

Hence, equation (s2) can be written as

$$\oiint_S \rho \mathbf{U}(\mathbf{U}\cdot\mathbf{n}) dA = -\oiint_S P\mathbf{n} dA. \tag{s7}$$

The surface area S of the control volume includes the inlet section A_1, the outlet section A_2, and the area of the walls A_w. The net outward momentum flux across the boundary of the control volume can be expressed as

$$\oiint_S \rho \mathbf{U}(\mathbf{U}\cdot\mathbf{n}) dA = \iint_{A_1} \rho \mathbf{U}_1(\mathbf{U}_1\cdot\mathbf{n}_1) dA + \iint_{A_w} \rho \mathbf{U}_w(\mathbf{U}_w\cdot\mathbf{n}_w) dA_w + \iint_{A_2} \rho \mathbf{U}_2(\mathbf{U}_2\cdot\mathbf{n}_2) dA, \tag{s8}$$

where $\mathbf{n}_i$, U_i and P_i are the outward unit normal vector, flow velocity, and the pressure to A_i, respectively. Since the wall A_w is non-penetrable surface, the flow velocity outward to A_w is zero: $\mathbf{U}_w \cdot \mathbf{n}_w = 0$. Then

$$\iint_{A_w} \rho \mathbf{U}_w(\mathbf{U}_w\cdot\mathbf{n}_w) dA_w = 0. \tag{s9}$$

For steady flow, equation (s8) can be written as

$$\oiint_S \rho\mathbf{U}(\mathbf{U}\cdot\mathbf{n})dA = -\rho\mathbf{U}_1U_1A_1 + \rho\mathbf{U}_2U_2A_2 = \rho U_1^2A_1\mathbf{n_1} + \rho U_2^2A_2\mathbf{n}_2 \,. \qquad \text{(s10)}$$

For the right part of equation (s7), the complete pressure force is the summation of the forces over the entire surface, yields:

$$-\oiint_S p\mathbf{n}dA = -\iint_{A_1} P_1\mathbf{n}_1 dA - \iint_{A_w} P\mathbf{n}_w dA - \iint_{A_2} P_2\mathbf{n}_2 dA \,. \qquad \text{(s11)}$$

The second term on the right is the pressure force from the surface of the groove of microjet. According to Newton's third law, the pressure force **F** acting on the microjet from the control volume can thus be expressed as

$$\mathbf{F} = \iint_{A_w} P\mathbf{n}dA \,. \qquad \text{(s12)}$$

If the pressure distribution is uniform, then (s11) can be rewritten as

$$-\oiint_S p\mathbf{n}dA = -P_1A_1\mathbf{n}_1 - P_2A_2\mathbf{n}_2 - \mathbf{F} \,. \qquad \text{(s13)}$$

Substitute equations (s10) and (s13) into equation (s7), then we get

$$\mathbf{F} = -(P_1 + \rho U_1^2)A_1\mathbf{n_1} - (P_2 + \rho U_2^2)A_2\mathbf{n}_2 \,. \qquad \text{(s14)}$$

For steady flow through the control volume:

$$\rho U_1A_1 = \rho U_2A_2 = const \text{, or } U_2 = \frac{A_1}{A_2}U_1 \,. \qquad \text{(s15)}$$

According to the Bernoulli equation,[S3,S6] for steady frictionless incompressible flow:

$$P_1 + \frac{\rho}{2}U_1^2 = P_2 + \frac{\rho}{2}U_2^2 = const \,. \qquad \text{(s16)}$$

Substitute equation (s15) into equation (s16), yields:

$$P_2 = P_1 + \frac{\rho}{2}(U_1^2 - U_2^2) = P_1 + \frac{\rho U_1^2}{2}[1 - (\frac{A_1}{A_2})^2] \,. \qquad \text{(s17)}$$

Substitute equations (s15) and (s17) into equation (s14), then the pressure force **F** can be expressed as

$$\begin{aligned}\mathbf{F} &= -(P_1 + \rho U_1^2)A_1\mathbf{n_1} - \{P_1 + \frac{\rho U_1^2}{2}[1 - (\frac{A_1}{A_2})^2] + \rho U_1^2(\frac{A_1}{A_2})^2\}A_2\mathbf{n}_2 \\ &= -(P_1 + \rho U_1^2)A_1\mathbf{n_1} - [P_1 + \frac{\rho U_1^2}{2} + \frac{\rho U_1^2}{2}(\frac{A_1}{A_2})^2]A_2\mathbf{n}_2\end{aligned} \,. \qquad \text{(s18)}$$

Neglecting the relative pressure of fluid P_1, equation (s18) can be rewritten as

$$\mathbf{F} = -\rho U_1^2A_1\mathbf{n_1} - \frac{\rho U_1^2}{2}[1 + (\frac{A_1}{A_2})^2]A_2\mathbf{n}_2 \,. \qquad \text{(s19)}$$

The unit normal vectors $\mathbf{n_1}$ and $\mathbf{n_2}$ can be expressed as

$$\begin{cases}\mathbf{n_1} = \mathbf{i}\cos\theta + \mathbf{j}\sin\theta \\ \mathbf{n}_2 = -\mathbf{i}\cos\theta - \mathbf{j}\sin\theta\end{cases} \,. \qquad \text{(s20)}$$

Thus,

$$\mathbf{F} = -\rho U_1^2 A_1(\mathbf{i}\cos\theta + \mathbf{j}\sin\theta) - \frac{\rho U_1^2}{2}[1 + (\frac{A_1}{A_2})^2]A_2(-\mathbf{i}\cos\theta - \mathbf{j}\sin\theta)\,. \quad \text{(s21)}$$

The component of the force acting on the groove with the direction perpendicular to the tube wall should be

$$F = -\rho U_1^2 A_1 \sin\theta + \frac{\rho U_1^2}{2}[1 + (\frac{A_1}{A_2})^2]A_2 \sin\theta\,. \quad \text{(s22)}$$

So far, we have calculated the force acting on one groove. However, as shown in Figure S2, there are several grooves on the wall of a grating-structured microjet. Here, γ (azimuth angle) is used to define the position of the groove, and the distribution of y component of the force along x direction can be written as

$$F_{y,\gamma} = -\rho U_1^2 A_1 \sin\theta\cos\gamma + \frac{\rho U_1^2}{2}[1 + (\frac{A_1}{A_2})^2]A_2 \sin\theta\cos\gamma\,. \quad \text{(s23)}$$

For all the grooves on the wall of a microjet:

$$F_y = \sum_{\gamma}\int_0^L F_{y,\gamma}dx\,. \quad \text{(s24)}$$

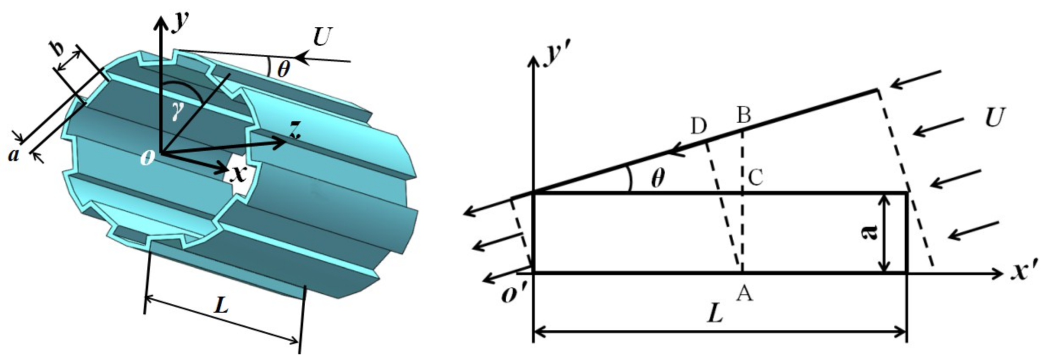
Here, the position of action point of the resultant force from the fluid pressure (x_p) is defined as

$$x_p = \frac{\sum_{\gamma}\int_0^L F_{y,\gamma}xdx}{F_y}, \quad \text{(s25)}$$

and the static stability margin (T) can be defined as

$$T = \frac{x_p - x_g}{L} \times 100\%, \quad \text{(s26)}$$

where x_g is position of the center of mass (see Figure S1).



(a) (b)

Figure S2. Schematic diagram of fluid field in a groove. (a) Geometrical parameters of a groove. (b) Profile of a streamtube.

In the following paragraphs, we will discuss how to calculate A_1 an A_2. The model and the parameters used are displayed in Figure S2a. a, b, and L are the depth, width, and length of the groove, respectively. Figure S2b shows the profile of a typical streamtube with γ=0°. The x′-axis is the bottom of the groove and the y′-axis is perpendicular to the bottom. θ is the angle between the direction of flow velocity and the x′-axis. At an arbitrary point A, we obtain

$$AD = AB \cdot \cos\theta\,, \tag{s27}$$

and

$$AB = AC + BC = a + x\tan\theta, \quad x \in (0, L)\,. \tag{s28}$$

The section area at an arbitrary point A (A_x) can be expressed as

$$A_x = b \cdot AD\,. \tag{s29}$$

Substitute equations (s27) and (s28) into equation (s29), then the section area A_x can be expressed as

$$A_x = b(a\cos\theta + x\sin\theta)\,. \tag{s30}$$

If $\gamma \neq 0°$, BC can be expressed as

$$BC = x\tan\theta \cdot \cos\gamma, \quad x \in (0, L), \gamma \in (-\frac{\pi}{2}, \frac{\pi}{2})\,. \tag{s31}$$

Then the section area at an arbitrary point in a groove with azimuth angle γ can be expressed as

$$A_x = b(a\cos\theta + x\sin\theta\cos\gamma)\,. \tag{s32}$$

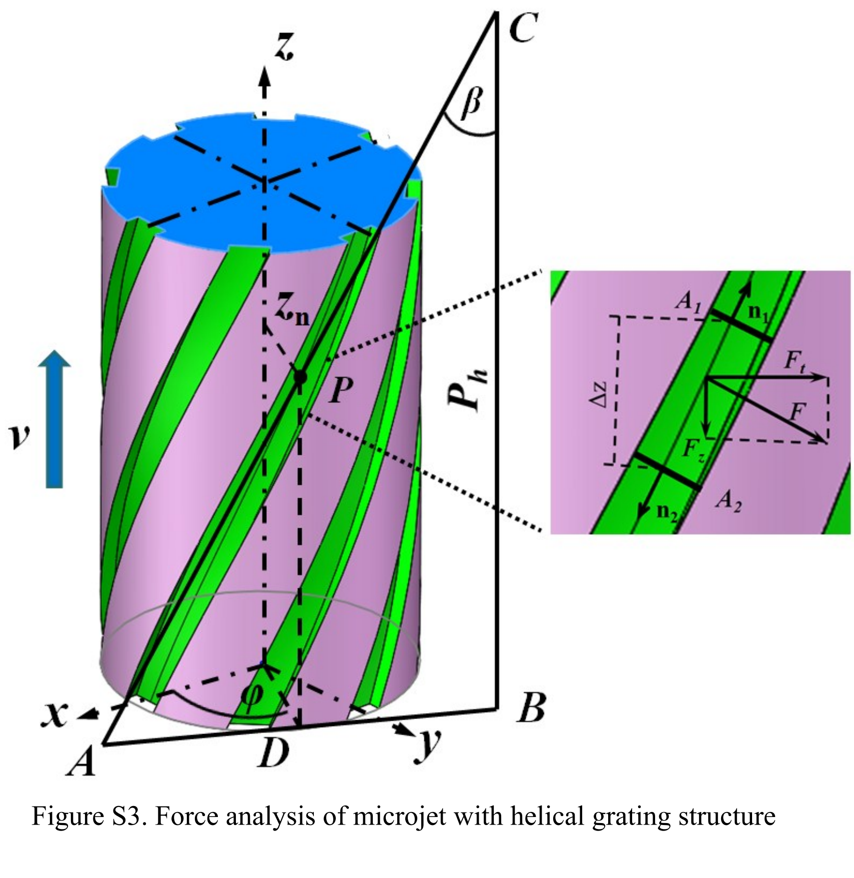


Figure S3. Force analysis of microjet with helical grating structure

2. The microjet with helical grating structure.

The force analysis of a microjet with helical grating structure (helical angle: β) is shown in Figure S3. △ABC is the expansion graph of the center line of a helical groove surface. P_h is the lead and AB is the circumference of the section perpendicular to the tube axis.

The fluid in the groove can also be regarded as a streamtube. Point D is the projection of an arbitrary point P on the center line of the groove surface. The angle ϕ between OD and x-axis is the azimuth angle of the point D. Then, the angle ϕ can be expressed as

$$\phi = \frac{\overline{AD}}{\pi D} * 2\pi = \frac{z \cdot \tan\beta}{R}. \tag{s33}$$

where z is the z-coordinate of point P.

The parameter equation of center line of the helical groove surface can be expressed as

$$\begin{cases} x = R\cos\phi \\ y = R\sin\phi \\ z = \phi R \cdot \mathrm{ctan}\,\beta \end{cases}, \tag{s34}$$

where R is the radius of the tubular microjet. The tangent of center line can thus be expressed as

$$\begin{cases} x' = -R\sin\phi \\ y' = R\cos\phi \\ z' = R \cdot \mathrm{ctan}\,\beta \end{cases}. \tag{s35}$$

Here, an arbitrary infinitesimal element with control volume at the height z in the streamtube is used for analysis. The inset of Figure S3 shows that the fluid flows through the control volume from the inlet section A_1 to outlet section A_2. The normal vectors of the inlet section A_1 and the outlet section A_2 are expressed as

$$\begin{cases} \mathbf{n_1} = -\mathbf{i} \cdot R\sin\beta\sin\phi_1 + \mathbf{j} \cdot R\sin\beta\cos\phi_1 + \mathbf{k} \cdot R \cdot \cos\beta \\ \mathbf{n_2} = \mathbf{i} \cdot R\sin\beta\sin\phi_2 - \mathbf{j} \cdot R\sin\beta\cos\phi_2 - \mathbf{k} \cdot R \cdot \cos\beta \end{cases}, \tag{s36}$$

where

$$\phi_2 = \frac{(z - \Delta z) \cdot \tan\beta}{R}. \tag{s37}$$

Substitute equation (s35) and A_1=A_2=A into equation (s19), then we get

$$\mathbf{F} = \rho U^2 AR\sin\beta[\mathbf{i}(\sin\phi_1 - \sin\phi_2) + \mathbf{j}(\cos\phi_2 - \cos\phi_1)]. \tag{s38}$$

Thus, the value of the pressure on the wall of the helical groove can be expressed as

$$F = \rho U^2 AR\sin\beta\sqrt{(\sin\phi_1 - \sin\phi_2)^2 + (\cos\phi_2 - \cos\phi_1)^2}. \tag{s39}$$

The distribution of its tangential component can be expressed as

$$F_t = F\cos\beta = \rho U^2 AR\sin\beta\cos\beta\sqrt{(\sin\phi_1 - \sin\phi_2)^2 + (\cos\phi_2 - \cos\phi_1)^2}\ . \quad \text{(s40)}$$

Then the resultant force acting on one groove from the fluid pressure can be expressed as

$$F_{tL} = \sum F_t = \int_0^L F_t dz\ . \quad \text{(s41)}$$

The moment of F_{tL} acting on the microjet should be

$$M_t = nF_{tL}R, \quad \text{(s42)}$$

where n is the number of the grooves.

As shown in Figure S4, a microjet with helical grating structure will be subjected to this moment when it moves along its axis in a flow field. Thus, it will rotate/spin around the axis.

Figure S4. Spin of a microjet with helical grating structure.

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