Electronic Supplementary Information (ESI)

Title: Polarization-independent broadband meta-holograms via polarization-dependent nanoholes

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1. The design of 2D polarization-independent hologram for arbitrary picture

![Target picture](image1)

![Phase distribution](image2)

![Amplitude distribution](image3)

![Holographic image](image4)

Figure S1: (a) The target picture of the letter “IOE” for the 2D polarization-independent hologram. (b) The phase and (c) amplitude distributions of the designed complex amplitude hologram. (d) The simulation holographic image of the designed polarization-independent hologram.
For designing polarization-independent 2D hologram of arbitrary target picture, we let the target picture in half of the frequency spectrum of the hologram to eliminate the interference between the target image and the conjugate image. As shown in Figure S1 (a), the letter “IOE” is located in top half of the frequency spectrum of the hologram. The last complex amplitude hologram is achieved by extracting the real part of the phase hologram \( e^{i\phi(x_0,y_0)} \) calculated by the IFTA and its phase and amplitude are shown in Figure S1 (b) and (c), respectively. The simulation holographic image of the designed complex amplitude when setting the incident light with linear polarization is shown in Figure S1 (d). The part of the holographic image in the green dashed line is the conjugate image of the preset target picture. It’s visually illustrated that half of the energy and frequency spectrum information are wasted as a result of the conjugate image. So the centrosymmetric snowflake picture is used as the target picture of the 2D hologram in the experiment, as demonstrated in the main text.

2. The theoretical analysis for different polarizations light incidence

We first analyze the case of the 2D hologram for noncenter-symmetric target. The incident light is assumed as a plane wave with the amplitude \( A_0 \). The target image intensity distribution is \( I(f_x,f_y) \) in the Fraunhofer region of the phase hologram (phase distribution \( \phi(x_0,y_0) \)), which is achieved by iterative Fourier-transform (IFTA) algorithm. Then we can obtain the relation:

\[
I(f_x,f_y)=A_0^2 \left| F(e^{i\phi(x_0,y_0)}) \right|^2
\]

(1)

\[
I(-f_x,-f_y)=A_0^2 \left| F(e^{-i\phi(x_0,y_0)}) \right|^2
\]

(2)

Where \((x_0,y_0)\) and \((f_x,f_y)\) are the coordinate of the fabricated hologram and its frequency spectrum, respectively. The \( F \) represents the Fourier transformation. The distribution of \( \cos(\phi(x_0,y_0)) \) is used as our polarization-independent complex amplitude hologram, which is achieved by superimposing the \( e^{i\phi(x_0,y_0)} \) and \( e^{-i\phi(x_0,y_0)} \). The complex amplitude-angle relation of \( \sin(2\theta) \) with x-polarization incident light is used to code the complex amplitude hologram. So the value of \( \theta \) is decided by:

\[
\theta = \frac{\pi}{4} \left( -\frac{\phi(x_0,y_0)}{2}, (0 < \phi(x_0,y_0) \leq \pi) \right)
\]

(3)

\[
\theta = -\frac{3\pi}{4} \left( -\frac{\phi(x_0,y_0)}{2}, (\pi < \phi(x_0,y_0) \leq 2\pi) \right)
\]

(4)

As x-polarization light impinges on the fabricated meta-hologram, the intensity distribution of the image plane is:

\[
I'(f_x,f_y)=A_0^2 \left| F(\sin(2\theta)) \right|^2 = A_0^2 \left| F(\cos(\phi(x_0,y_0))) \right|^2
\]

\[
= 0.25A_0^2 \left[ F(e^{i\phi(x_0,y_0)}) + F(e^{-i\phi(x_0,y_0)}) \right]^2
\]

\[
= 0.25 \left[ I(f_x,f_y) + I(-f_x,-f_y) + A_0^2 F(e^{i\phi(x_0,y_0)}) F(e^{-i\phi(x_0,y_0)})^* + A_0^2 F(e^{i\phi(x_0,y_0)})^* F(e^{-i\phi(x_0,y_0)}) \right]
\]

Theoretically, the value of the third item of equation (5), \( 0.25A_0^2 F(e^{i\phi(x_0,y_0)}) F(e^{-i\phi(x_0,y_0)})^* \), is 0 because of the target image, coming from \( F(e^{i\phi(x_0,y_0)}) \), is located in half of the frequency spectrum of the hologram and its conjugate image, coming from \( F(e^{-i\phi(x_0,y_0)})^* \).
holographic image of the depolarized light is much dimmer than the results with x-polarization incidence. Nevertheless, the holographic result is the superimposition of those images. If the x-polarization is changed into y-polarization, then:

\[ I''(f_x, f_y) = A_0^2 \left| F(e^{i2\phi}) \right|^2 = A_0^2 \left| F(-e^{i2\phi}) \right|^2 = I'(f_x, f_y) \]  

(6)

From equation (6), we can conclude that the holographic image achieved with y-polarization incident light is the same for x-polarization. In theory, a certain polarization is LCP, thus, the image intensity can be expressed as:

\[ I''(f_x, f_y) = A_0^2 \left| F(e^{i2\theta/2}) \right|^2 = A_0^2 \left| F(-e^{i2\theta/2}) \right|^2 \approx \frac{1}{4} \left( -f_x, -f_y \right) \text{ if } (0 < \phi(x_0, y_0) \leq \pi) \]

\[ = A_0^2 \left| F(e^{i2\theta/2}) \right|^2 \approx \frac{1}{4} \left( f_x, f_y \right) \text{ if } (\pi < \phi(x_0, y_0) \leq 2\pi) \]  

(7)

The quantity of the \( \phi(x_0, y_0) \) value in the range from 0 to \( \pi \) is approximately equal to that in the range of \( \pi \) to \( 2\pi \). Therefore, the amplitudes of the target image and conjugate image of polarization-independent hologram are about 1/2 time to the original amplitude of the holographic image of the IFTA. So the intensity coefficient (1/2)\(^2\)=1/4 would be achieved, as shown in equation (7). This equation indicates the target holographic image can be achieved with LCP light incidence.

While the incident polarization is changed into RCP, the image intensity becomes:

\[ I''(f_x, f_y) = A_0^2 \left| F(e^{-i2\phi}) \right|^2 = A_0^2 \left| F(e^{-i2\theta/2}) \right|^2 \approx \frac{1}{4} \left( f_x, f_y \right) \text{ if } (0 < \phi(x_0, y_0) \leq \pi) \]

\[ = A_0^2 \left| F(e^{-i2\theta/2}) \right|^2 \approx \frac{1}{4} \left( -f_x, -f_y \right) \text{ if } (\pi < \phi(x_0, y_0) \leq 2\pi) \]  

(8)

We can conclude from equations (7)-(8) that the target image \( I(f_x, f_y) \) can be implemented with both LCP and RCP incident light even though half of the energy is
sacrificed for the conjugate image. With LCP incident light, the target holographic image is achieved with the pixels with phase $\phi(x_0, y_0)$ in the range from $\pi$ to $2\pi$, while the corresponding image is realized using the phase $\phi(x_0, y_0)$ in the range from 0 to $\pi$ with RCP incident light. That is to say, the target image intensity of LCP and RCP incident light are not entirely equal. However, the discrepancy is very small and can be neglected since the pixel quantity in those two ranges is approximately equal, especially when the hologram contains abundant of pixels. For example, the intensity discrepancy of the target images with LCP and RCP light incidence for the design shown in Section 1 of ESI is about 0.00012 time to the total intensity of target image. So little discrepancy cannot be measured in our experiment. In addition, the profiles of the target holographic images are always identical. The theoretical analysis shown in equations (5)-(8) indicates the polarization-independent character of the 2D meta-hologram designed by our proposed algorithm. The theoretical analysis of the 3D polarization-independent hologram is similar to the case of 2D hologram, so we don’t show it here again.

3. The experimental results of 2D hologram for different polarizations

![Figure S2: The experimental holographic image for incident (a) LCP, (b) RCP, (c) 30° LEP, (d) 30° REP, (e) 30° linear polarization and (f) 45° linear polarization light. LCP: left circular polarization, RCP: right circular polarization, LEP: left ellipse polarization, REP: right ellipse polarization.](image)

For further verifying the polarization-independent property of our fabricated sample, other different polarizations light, such as LCP (left circular polarization), RCP (right circular polarization), LEP (left ellipse polarization), REP (right ellipse polarization) and linear polarization light, is used to illuminate the meta-hologram. The corresponding experimental results for LCP, RCP, 30°LEP and 30°REP light are shown in Figure S2 (a)-(d), respectively. The related angles for the 30° are marked in
Figure S2 (c) and (d), which means the long axis length is $\sqrt{3}$ times to short axis length of the polarization trajectory ellipse. It should be noted that the LP$_2$ (Figure 4e) is removed and a QWP$_2$ (quarter wave plate) is added between the sample and LP$_3$ in the experimental setup, which is similar to the experiment setup shown in Figure 3e. The different CP (circular polarization) and EP (ellipse polarization) light are achieved by rotating the orientation angle of the QWP and the noise is filtered by the pair of QWP$_2$ and LP$_3$. Figure S2 (e) and (f) show the holographic images for linear polarization light, whose orientation angles between the polarization orientation and vertical direction are 30° and 45°, respectively. The experimental results shown in Figure S2 further prove the feasible of the proposed algorithm for realizing the polarization-independent holographic image.

4. The results of another design of 3D polarization-independent hologram

Figure S3: (a) Schematic diagram of the 3D target object, which is consisted of
sixteen points in different position. The phase (b) and amplitude (c) distribution of the
designed hologram (100 × 100 pixels, 20μm × 20μm). The coding structure is shown
in the inset of Figure (b) with the size: P=200nm (period), L=140nm (length),
W=60nm (width). θ: the orientation angle of the elongated aperture. The thickness of
the gold film is 120nm. The substrate is SiO$_2$. (d)-(g) The simulation results of the
holographic image with different distances in the z-axis with x-polarization light
incidence using CST software. (h)-(k) The corresponding simulation results with y-
polarization light incidence. The incident wavelength is set with 405nm.

For further verifying the feasibility of the proposed algorithm for 3D image, we
design another 3D hologram for the target object shown in Figure S3 (a). The target
object is consisted of sixteen points with different coordinates in x/y/z-axis. We firstly
use the point source algorithm$^1, 2$ to achieve the phase-only hologram (phase
distribution $\phi(x_0,y_0)$) and the polarization-independent complex amplitude hologram
$\cos(\phi(x_0,y_0))$ is obtained by extracting the real part of $e^{i\phi(x_0,y_0)}$. The phase and
amplitude distributions of the designed 3D polarization-independent hologram are
shown in Figure S3 (b)-(c). The elongated aperture used to code the hologram is
shown in the inset of Figure S3 (b). The period (P=200nm), length (L=140nm) and
width (W=60nm) are shown in the figure. The phase and amplitude of the hologram
are controlled by changing the orientation angle ($\theta$) with the amplitude-angle relation
$|\sin(2\theta)|$ and phase-angle relation. More information about the used structure can be
found in other works.$^2-4$ The reason for using the elongated aperture with period
200nm other than elliptical nanohole with period 350nm is decreasing the
computation load with the same pixels in the CST Microwave Studios. The open
boundary conditions are used along x, y and z directions in the full model simulation
with CST software. The electromagnetic field distribution on the plane with a distance
0.3μm away from the metasurface is calculate by CST software and the electric field
distribution on other planes are simulated by using Fresnel diffraction theory. The
simulation holographic images of several transections with different distance along z-
axis with x-polarization and y-polarization incidence are shown in Figure S3 (d)-(g)
and (h)-(k), respectively. With 2D results on different plane, the 3D feature of the 3D
image is verified. The holographic images shown Figure S3 (d)-(g) match well with the
results shown in Figure S3 (h)-(k), which further demonstrated the proposed
algorithm is feasible for designing real 3D polarization-independent hologram.

2013, 4, 2808.
2. X. Zhang, J. Jin, M. Pu, X. Li, X. Ma, P. Gao, Z. Zhao, Y. Wang, C. Wang
and X. Luo, Nanoscale, 2016, 9, 1409-1415.
3. L. Liu, X. Zhang, M. Kenney, X. Su, N. Xu, C. Ouyang, Y. Shi, J. Han, W.
4. D. Tang, C. Wang, Z. Zhao, Y. Wang, M. Pu, X. Li, P. Gao and X. Luo, Laser