Supporting Information

Tuning the transport and magnetism in Cr-Bi\textsubscript{2}Se\textsubscript{3} Topological Insulator by Sb doping\textsuperscript{†}

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Figure S1

Figure S1 (a)-(d) display the RHEED pattern of various samples and (e)-(h) show the corresponding surface morphology that was analyzed by AFM. Without Cr
and Sb doping, the Bi₂Se₃ shows normal scale terraces. The surface of each films consists of characteristic triangular-shaped terraces and each roughness is around 1nm, reflecting the crystal structure still remains a fully tradition appearance of Bi₂Se₃. The width of the terraces decreases with increasing the doping of Sb and Cr. While codoped Sb and Cr samples exhibit surface roughness \( R_q \sim 0.912 \) nm. The obvious reduction of terrace width may indicate the evidence of Cr and Sb substitution into Bi site.

**Figure S2**

Sheet resistance \( R_s \) measured at 2K using physical property measurement system (PPMS).

<table>
<thead>
<tr>
<th>x in Cr-doped ((\text{Bi}, \text{Sb})_x \text{Se}_3)</th>
<th>( R_s ) (Ohm)</th>
<th>( k_f )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1478.3</td>
<td>~17</td>
</tr>
<tr>
<td>0.35</td>
<td>13715.9</td>
<td>~2</td>
</tr>
</tbody>
</table>

**Figure S3**

Fitting of MC curves using Hikami-Larkin-Nagoka formula:

\[
\Delta \sigma = \alpha \frac{e^2}{\pi \hbar} \left[ \psi \left( \frac{1}{2} + \frac{B \varphi}{B} \right) - \ln \left( \frac{B \varphi}{B} \right) \right]
\]

where \( \alpha \) is a coefficient indicating the type of localization (\( \alpha \) is positive for weak localization and negative for weak anti-localization), \( \psi \) is digamma function, and \( B \varphi = \hbar/4e l^2_p \) is characteristic field with \( l^2_p = (D \tau_p)^{1/2} \) is the phase coherent length, \( \tau_p \) is the phase coherent time and \( D \) is diffusion constant.
Figure S3 (a) The MC curves of Bi$_2$Se$_3$, (Bi$_{0.65}$Sb$_{0.35}$)$_2$Se$_3$, (Bi$_{0.93}$Cr$_{0.07}$)$_2$Se$_3$ and (Bi$_{0.58}$Cr$_{0.07}$Sb$_{0.35}$)$_2$Se$_3$ are fitted with HLN equation; (b) & (c) illustrate the extracted $\alpha$ and $l_\rho$.

Figure S4
The out-of-plane magnetic properties of (Cr, Bi)$_2$Se$_3$ (Cr-BS) and Sb-doped (Cr, Bi)$_2$Se$_3$ (Cr-BSS) were measured by SQUID magnetometer at 2K. It’s difficult to resolve the magnetic signal owing to the small amount of Cr doping level ($y=0.07$), Thus, for this SQUID measurement, we used thicker sample (80 QLs) with higher Cr concentration than that shown in the manuscript.

As shown in Fig. S4, both Cr-BS and Cr-BSS exhibit nonlinear magnetization as a function of magnetic fields. However, Sb-doped Cr-BS clearly presents significant larger magnetic moment than that of non-Sb-doped case. Such nonlinear curve reveals similar features as AHE, which confirms the non-linear Hall effect indeed induced by the magnetism.

Although the used samples are different from those in the manuscript, with this experiments, we demonstrate the positive role of Sb-doping in enhancing the magnetism in Cr-BS at a fix Cr content.
Figure S4 M-H loop of the 80QL (Cr, Bi)$_2$Se$_3$ and Sb-doped (Cr, Bi)$_2$Se$_3$ measured at B-field applied perpendicular to sample plane at 2K.