Electronic Supplementary Information for *Soft Matter* manuscript: Structural behavior of competitive temperature and pH-responsive tethered polymer layers

Simona Morochnik[a], Rikkert J. Nap[b], Guillermo A. Ameer[a], and Igal Szleifer[*b]

[a] *Department of Biological Sciences, Department of Biomedical Engineering, Chemistry of Life Processes Institute, Northwestern University, Evanston, Illinois, USA.*

[b] *Department of Biomedical Engineering, Department of Chemistry, Chemistry of Life Processes Institute, Northwestern University, Evanston, Illinois, USA, email: igalsz@northwestern.edu*

I. Expanded theoretical approach

From the total Helmholtz free energy,

$$F = -TS_{conf} + F_{chm} + F_{HB} - TS_{mix} + F_{elect} + U_{vdW} \quad (1)$$

the latter three terms are expanded as follows: the fourth term in eq. 1 is the mixing entropy of free species in solution and given by

$$-\frac{S_{mix}}{k_B A} = \sum_\gamma \int dz \rho_\gamma(z)\left[\ln\left(\rho_\gamma(z) v_w\right) - 1\right] \quad (S.1)$$

where $\rho_\gamma(z)$ is the location-dependent density of the mobile species. The index γ runs over water, Na^+, Cl^-, H^+, and OH^- molecules and v_w is the volume of water. The tethered polymer is immersed in an aqueous solution that is in chemical equilibrium with a reservoir of given salt concentration and pH. The Na^+ and Cl^- ions are assumed to be completely dissociated.

The fifth term in the free energy (eq. 1) is the electrostatic energy functional

$$\frac{\beta F_{elect}}{A} = \beta \int dz \left[\langle \rho_q(z) \rangle \Psi(z) - \frac{1}{2}\varepsilon_0 \varepsilon_w \left(\frac{\mathrm{d}\Psi}{\mathrm{dz}}\right)^2\right] \quad (S.2)$$

where $\Psi(z)$ is the electrostatic potential, ε_0 is the absolute permittivity of vacuum and ε_w is the relative dielectric permittivity of the water whose exact temperature dependence is given by $\varepsilon_w = (3.70886\,x\,10^4 - 8.2168\,x\,10^1(T))\,/\,4.21854\,x\,10^2 + T$. $\langle \rho_q(z) \rangle$ is the ensemble average of the charge density at z and given by

$$\langle \rho_q(z) \rangle = f_d(z) \langle \rho_A(z) \rangle q_A + \sum_k \rho_k(z)\, q_k \quad (S.3)$$

where q_A is the electric charge of the deprotonated A monomer and the second term is the sum over the charges of all free species, k = Na^+, Cl^-, H^+, and OH^-.

The sixth term in eq. 1 describes the effective Van der Waals interaction energy between monomers

$$\beta U_{vdW} = -\frac{\chi_A}{2v_w}\int dz\ \langle \phi_A(z) \rangle^2 - \frac{\chi_B}{2v_w}\int dz\ \langle \phi_B(z) \rangle^2 \quad (S.4)$$

where χ_i is the effective interaction parameter that measures the strength of attraction between like-monomers. Monomers are either pH and temperature responsive (A) or hydrophobic (B). Here, χ_i is defined as $\chi_i = \varepsilon_i \frac{T_{ref}}{T}$ where T_{ref} is 298 K and T is the input temperature. As T increases, χ_i will decrease, reducing the strength of attractions. The interactions lead to phase separation characteristic of expected, UCST (upper critical solution temperature) systems. Because the B monomers are neutral and hydrophobic, this interaction suffices to describe their phase behavior. However, PEG exhibits inverse solubility, or LCST behavior, which is due to the formation of hydrogen bond pairs between polymer-water and water-water. Thus, we must include another term in the free energy to describe the hydrogen bond formation.

The sixth term is the associative free energy of hydrogen bonding where the formula is given in the main text. It builds on the work of Dormidontova for PEG solutions which was applied by Ren et al. to PEG tethered layers. Here, $x_P(z)$ is the local fraction of polymer-water hydrogen bonds for A monomers and $x_w(z)$ is the local fraction of water-water hydrogen bonds. They are defined as

$$x_p(z) = \frac{n_A(z)}{2N_A(z)} \text{ and } x_w(z) = \frac{n_w(z)}{2N_w(z)}$$

where $n_A(z)$ and $n_w(z)$ is the number of hydrogen bonds between A units and water, and between water and water, respectively. Because we assume each monomer can form 2 hydrogen bonds, $2N_A$ represents the total number of feasible monomer-water hydrogen bond pairs. Likewise, each oxygen of water can also form 2 hydrogen bonds. So, $2N_w$ is the total number of potential water-water hydrogen bond pairs where N_w is the number of water molecules.

In eq. 6 the first line corresponds to the mixing entropy contribution associated with forming monomer-water hydrogen bond pairs plus the free energy gain associated with the formation of this bond (ΔF_P). The next line is identical and describes the mixing entropy associated with having a water-water hydrogen bond pair plus the free energy gain associated with the formation of this bond (ΔF_w). The last lines contain a necessary correction for the change that results upon formation of a hydrogen bond, as it reduces the number of available hydrogen bond sites.

Finally, the repulsive interactions between the polymer and solvent are represented through excluded volume interactions. The intra-chain interactions are considered exactly during generation of the chain conformations. This is because all chain conformations are self-avoiding internally and with the surface. The inter-molecular repulsions are accounted for by assuming the system to be incompressible at every position. This is given by

$$1 = \langle \rho_p(z) \rangle + \sum_\gamma \rho_\gamma(z) v_\gamma \text{ (S.5)}$$

where γ =water, Na$^+$, Cl$^-$, H$^+$, and OH$^-$. This constraint is enforced by the introduction of Lagrange multipliers. Here v_γ is the volume of the mobile species.

Thus, the total Helmholtz free energy becomes

$$\begin{aligned}
\beta \frac{F}{A} = \sigma \sum_\alpha P(\alpha) \ln P(\alpha) + \sum_\gamma \int dz \rho_\gamma(z) \left[\ln\left(\rho_\gamma(z) v_w\right) - 1 \right] - \frac{\chi_A}{2v_w} \int dz \ \langle \phi_A(z) \rangle^2 - \frac{\chi_B}{2v_w} \int dz \ \langle \phi_B(z) \rangle^2 \\
+ \int dz \, \langle \rho_A(z) \rangle f_d(z) (\ln f_d(z) + \beta \mu_{A^-}^0) + \int dz \, \langle \rho_A(z) \rangle \left(1 - f_d(z)\right) \left[\ln\left(1 - f_d(z)\right) + \beta \mu_{AH}^0 \right] \\
+ \int dz \left(\rho_{H^+}(z) \beta \mu_{H^+}^0 + \rho_{OH^-}(z) \beta \mu_{OH^-}^0 \right) + \int dz \left[\langle \rho_q(z) \rangle \beta \Psi(z) - \frac{1}{2} \beta \varepsilon_0 \varepsilon_w \left(\nabla \Psi(z) \right)^2 \right] \\
+ \int dz \, 2 \langle \rho_A(z) \rangle \left[x_P(z) \ln x_P(z) + \left(1 - x_P(z)\right) \ln\left(1 - x_P(z)\right) + x_P(z) \beta \Delta F_P \right)] \\
+ \int dz \, 2 \rho_w(z) \left[x_w(z) \ln x_w(z) + \left(1 - x_w(z)\right) \ln\left(1 - x_w(z)\right) + x_w(z) \beta \Delta F_w \right)] \\
+ \int dz 2 \rho_w(z) \left[1 - x_w(z) - x_p(z) \frac{\langle \rho_A(z) \rangle}{\rho_w(z)} \right] \ln \left[1 - x_w(z) - x_p(z) \frac{\langle \rho_A(z) \rangle}{\rho_w(z)} \right] \\
- \int dz 2 \rho_w(z) \left[x_w(z) + x_p(z) \frac{\langle \rho_A(z) \rangle}{\rho_w(z)} \right] \ln \frac{2 \rho_w(z) v_w}{e}
\end{aligned}$$

$$\text{(S.6)}$$

The total free energy is minimized under the constraints of incompressibility and the condition that the system is in contact with a bath of anions, cations, protons and hydroxyls. This yields the following semi-grand thermodynamic potential, W

$$W = F - \sum_{\gamma = +,-,OH-} \beta\mu_\gamma \int dz\, \rho_\gamma(z) - \beta\mu_{H^+}\left(\int dz\, \rho_{H^+}(z) + \big(1 - f_d(z)\big)\langle\rho_A(z)\rangle\right)$$
$$+\beta \int dz\, \pi(z)\big(\phi_p(z) + \left(\textstyle\sum_\gamma \phi_\gamma(z)\right) - 1\big) \quad (S.7)$$

W is minimized with respect to $P(\alpha)$, $\rho_\gamma(z)$, $f_d(z)$, $x_w(z)$ and $x_p(z)$ and varied with respect to the electrostatic potential $\Psi(z)$. Minimization with respect to $\rho_\gamma(z)$ gives

$$\rho_\gamma(z) = \frac{e^{\beta(\mu_\gamma^0 - \mu_\gamma)}}{v_w} \exp\big(-\beta\pi(z)v_\gamma - \beta\Psi(z)q_\gamma\big) \quad (S.8)$$

and minimizing with respect to the local degree of dissociation $f_d(z)$ yields

$$\frac{f_d}{1-f_d} = \frac{K_a^0\, e^{-\beta\pi(z)\Delta v_a}}{v_w \rho_{H+}} \quad (S.9)$$

where $K_a^0 = e^{-\beta\Delta G_a^0}$ is the standard chemical equilibrium constant and $G_a^0 = \mu_{A^-}^0 + \mu_{H^+}^0 - \mu_{AH}^0$ is the standard free energy change of the reaction $AH \leftrightarrow A^- + H^+$. K_a^0 is proportional to the equilibrium constant of the acid-base reaction, K_a , which expressed in molar units is

$$K_a = \frac{[A^-][H^+]}{[AH]}$$

and the relationship between this experimental K_a and the theoretical K_a^0 is expressed as

$$K_a^0 = \frac{K_a N_a v_w}{(1 - x_w)^4}$$

Minimizing with respect to the water volume fraction is

$$\phi_w(z) = \exp\left[-\beta\pi(z)v_w - 2\ln\big(1 - x_w(z)\big) - 2\ln\left(1 - x_w(z) - x_p(z)\frac{\rho_A(z)}{\rho_w(z)}\right)\right]$$
$$(S.10)$$

and minimizing with respect to the local fraction of polymer-water $x_p(z)$ and water-water hydrogen bonds $x_w(z)$ results in

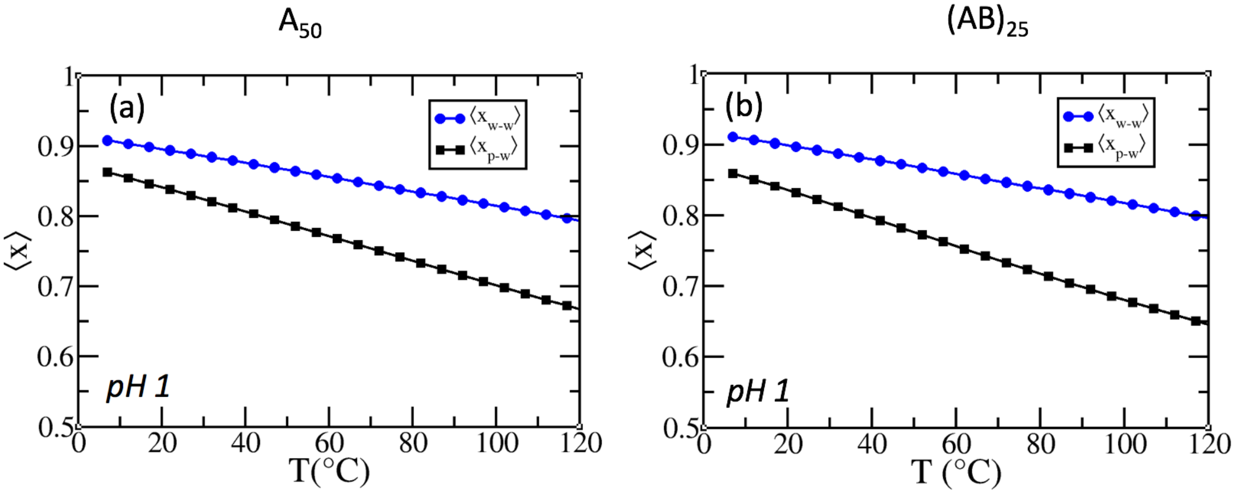
$$x_p(z) = 2exp(\beta\Delta F_P)\left(1 - x_p(z)\right)(\langle\phi_w(z)\rangle)\left(1 - x_w(z) - x_p(z)\frac{\rho_A(z)}{\rho_w(z)}\right) \quad (S.11)$$
$$x_w(z) = 2exp(\beta\Delta F_w)\big(1 - x_w(z)\big)(\langle\phi_w(z)\rangle)\left(1 - x_w(z) - x_p(z)\frac{\rho_A(z)}{\rho_w(z)}\right) \quad (S.12)$$

Finally, the variation of the free energy with respect to the electrostatic potential gives the Poisson equation and its boundary condition and is given by

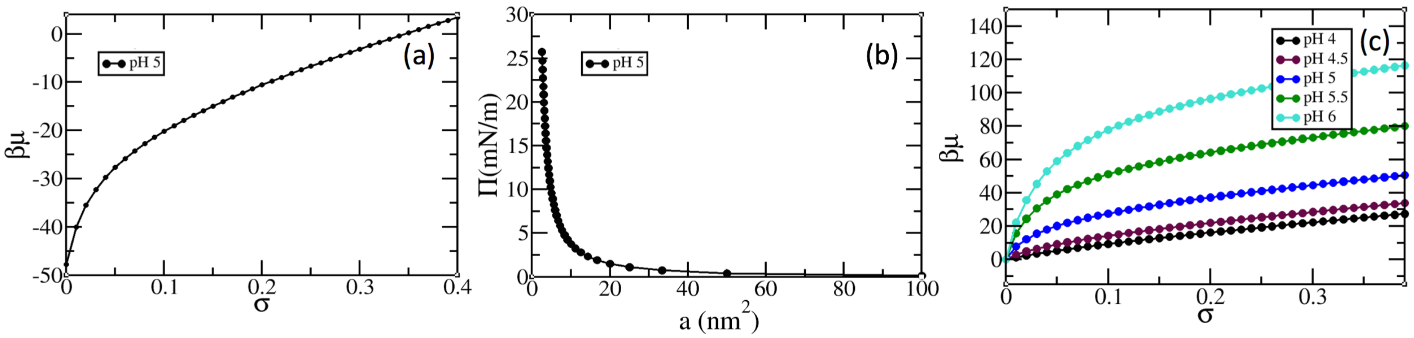
$$\varepsilon{\nabla_z}^2\Psi(z) = -\langle\rho_q(z)\rangle\ , \quad \left.\frac{d\Psi}{dz}\right|_{z=0} = 0 \quad (S.13)$$

The electrostatic potential is influenced by the local polymer and counter-ion densities thus there is a coupling between the Poisson equation and these terms.

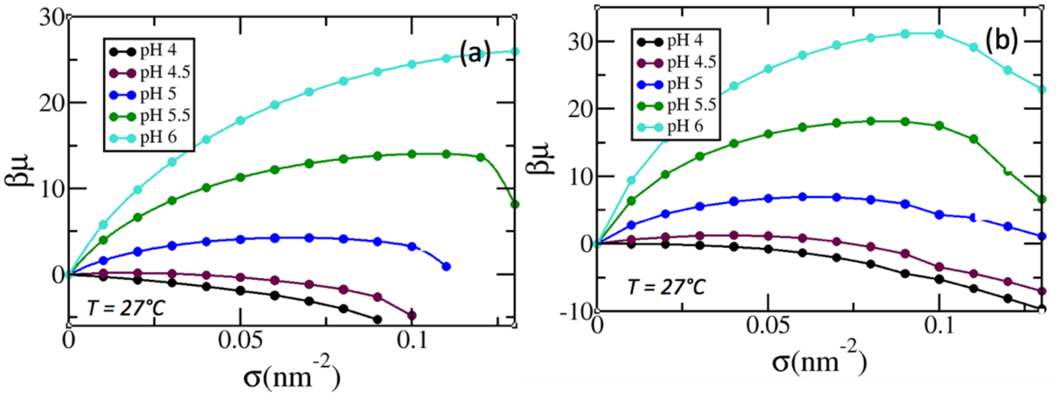
II. Supplementary Figures



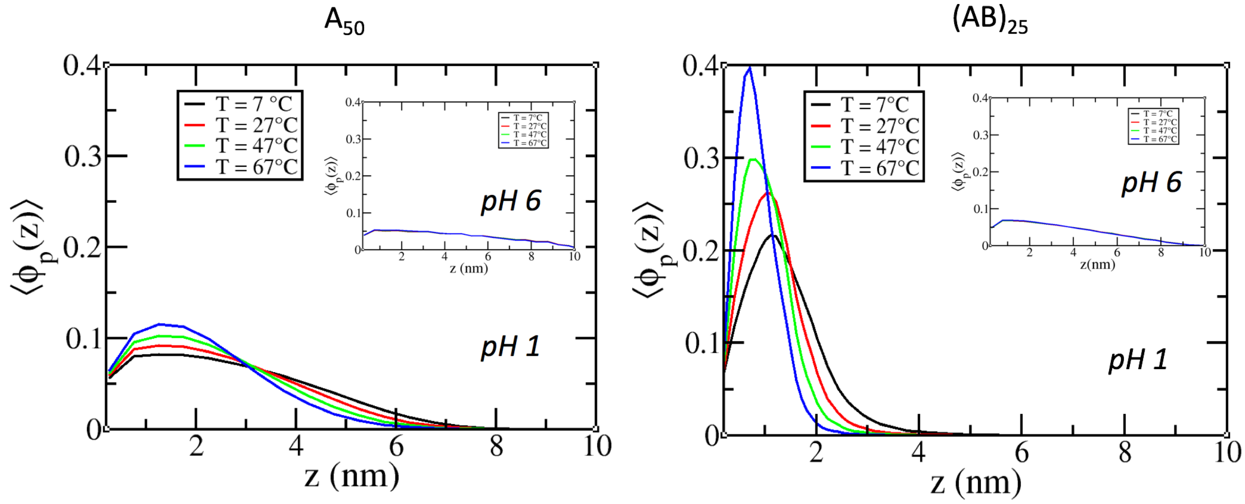
S1. Average fraction of polymer-water and water-water hydrogen bonds for A_{50} (a) and alternating $(AB)_{25}$ (b) cases shown as a function of increasing temperature. As p-w hydrogen bonds decrease, the polymer is excluded from the solvent and urged toward collapse. The rate of decrease in p-w and w-w hydrogen bonds is dependent on the respective entropic and enthalpic values of hydrogen bond formation for either PEG or water.



S2. Thermodynamic properties of A_{50} system. Chemical potential of polymers shown as a function of surface coverage for responsive polymers for pH 5 at 27°C (S2a). The positive slope of the chemical potential corresponds to a thermodynamically stable, homogenous system. The osmotic pressure associated with pH 5 shows the expected system—as polymer density increases, the monomers are pushed closer together generating more interpolymer repulsions. (S2b). The chemical potential is also shown for wider pH range (S2c). The polymers are 50 segments in length. The surface coverage is σ = 0.1 chains/nm^2 and the salt concentration is 0.1 M.



S3. Chemical potential plotted against surface coverage for diblock polymers at room temperature for either $B_{25}A_{25}$ (a) or $A_{25}B_{25}$ (b). The salt concentration is 0.1 M. Chemical potentials were shifted to zero.



S4. Average polymer volume fraction plotted as a function of distance from surface with ε = 0.67. This value matches the experimental value used by Dormidontova. We use ε = 0.1 in calculations for ease of computation. However, we show here that the difference between the two values is only in the degree of collapse as the strength of attraction is increased. The overall behavioral trend is similar.