Supporting Information for:

Effects of Vertical Hydrodynamic Mixing on Photomineralization of Dissolved Organic Carbon in Arctic Surface Waters

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20 pages; 4 tables; 5 figures
Figure S1. Site map of the Kuparuk River on the North Slope of Alaska. Photochemical parameters were measured from water samples collected from the sampling sites (red circles). River discharge and channel geometry data were collected from 8 gauging stations along the Kuparuk River (blue crosses).

Figure S2. Summary of (A) photochemical parameters (years 2011-2013) as a function of light wavelength and (B) hydrology parameters (years 2013-2015) as a function of downstream distance in the Kuparuk River. Panel (A) contains spectra of photon flux at the water surface ($Q_\lambda$) from 19-June-2013, a sunny (clear sky) day at Toolik Field Station in Arctic Alaska, with different colors denoting different hours within a day, ±95% confidence interval of the light attenuation coefficient of CDOM ($a_{CDOM,\lambda}$) and the photo-lability of CDOM ($\Phi_\lambda$), and parameters $\alpha_\lambda$ and $\epsilon_\lambda$ that relate DOC concentration to $a_{CDOM,\lambda}$. Panel (B) contains water surface slope ($S$), and ranges of water column depth ($H$), river width ($W$), and flow discharge ($Q$).

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Figure S4. Water column depth ($H$) and water surface slope ($S$) as a function of flow discharge ($Q$) for arctic stream, rivers, and beaded streams. Data sources are summarized in Table S1. Dash-dot lines represent 95% prediction intervals of the power law relationships of $H$-$Q$ and $S$-$Q$.

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Figure S3. Spectra of light attenuation coefficient of CDOM (aCDOM$_\lambda$) and photo-lability of CDOM (Φ$_\lambda$) for arctic streams, rivers, and lakes$^{1,7}$. Parameters aCDOM$_\lambda$ and Φ$_\lambda$ exhibit an exponential relationship with wavelength, with decadic slopes of -0.005 and -0.009, respectively. These slopes are based on log10 of aCDOM$_\lambda$ and Φ$_\lambda$, which is the same as exponential slopes -0.012 and -0.021 for natural log of aCDOM$_\lambda$ and Φ$_\lambda$. 
Figure S4. Water column depth (H) and water surface slope (S) as a function of flow discharge (Q) for arctic stream, rivers, and beaded streams. Data sources are summarized in Table S1. Dash-dot lines represent 95% prediction intervals of the power law relationships of H-Q and S-Q.
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SI-1. No mixing limitation when CDOM is the only light attenuating constituent.

When CDOM is the only light attenuating constituent, light does not necessarily decay exponentially with depth, especially when CDOM is nonhomogeneously distributed. At each vertical position (y) in the water column, photon flux \( Q_\lambda(y) \) depends on the total amount of CDOM above that position\(^8\).

\[
Q_\lambda(y) = Q_{dso-\lambda} e^{\int_y^0 aCDOM_\lambda(y)dy}
\]  \hspace{1cm} (S1)

where \( Q_{dso-\lambda} \) is the photon flux at the water surface, \( aCDOM_\lambda(y) \) is the light attenuation coefficient of CDOM, and \( \lambda \) is wavelength. The photomineralization rate profile \( (PM(y)) \) is the product of the photo-lability of CDOM to photomineralization \( (\Phi_\lambda) \), the light attenuation coefficient of CDOM, and the photon flux\(^1\).

\[
PM(y) = \sum_\lambda \phi_\lambda aCDOM_\lambda(y) Q_{dso-\lambda} e^{\int_y^0 aCDOM_\lambda(y)dy}
\]  \hspace{1cm} (S2)

Using integration by parts and calculus theorem, the total attenuation of light across the water column \( \Delta Q_\lambda \) can be derived as

\[
\Delta Q_\lambda = Q_{dso-\lambda} - Q_{dso-\lambda} e^{\int_y^0 aCDOM_\lambda(y)dy}
\]  \hspace{1cm} (S3)

where the exponential term on the right-hand side of (S3) is a function of the total CDOM amount in the water column, so the total attenuation of light across the water column is the same regardless of the distribution of CDOM. Similarly, the upscaled photomineralization rate over the water column can be derived as

\[
\int_0^H PM(y)dy = \phi_\lambda Q_{dso-\lambda} \left(1 - e^{\int_y^0 aCDOM_\lambda(y)dy}\right)
\]  \hspace{1cm} (S4)

which is independent of the distribution of CDOM. Therefore, in arctic waters where CDOM is the only constituent that attenuates light, upscaled photomineralization rate over the water column is not limited by vertical hydrodynamic mixing.

SI-2. Compiling photochemical and hydrological data of arctic waters.

Spectra of light attenuation coefficient of CDOM \( (aCDOM_\lambda) \) and photo-lability of CDOM \( (\Phi_\lambda) \) are available from over 1000 samples for arctic streams and rivers and over 2000 samples for arctic lakes\(^,\)\(^7\). These spectra show exponential relationships with wavelength \( (\lambda) \) and\(^8\)\(^9\)\(^10\)\(^11\). In latter cases, full spectra were extrapolated by assuming the same decadic slopes (-0.005 for \( aCDOM_\lambda \) and -0.009 for \( \Phi_\lambda \) with wavelength as the available spectra \( (\text{Figure S3}) \). Note that these slopes are based on \( \log_{10} \) of \( aCDOM_\lambda \) and \( \Phi_\lambda \), which is the same as exponential slopes -0.012 and -0.021 for commonly reported natural log \( aCDOM_\lambda \) \(^8\)\(^9\)\(^10\)\(^11\) and \( \Phi_\lambda \). We also assumed that, within each type of system, high DOC concentration corresponds to high \( aCDOM_\lambda \) and low DOC concentration corresponds to low \( aCDOM_\lambda \), due to the fact that DOC concentration and \( aCDOM_\lambda \) were usually not reported together.

In streams and rivers, water surface slope \( (S) \) and water column depth \( (H) \) are expected to co-vary, and both are a function of flow discharge \( (Q) \), although from the literature \( S, H, \) and \( Q \) were usually not reported together. In order to predict \( S \) and \( H \) from reported values while capturing their covariation, we plotted \( S-Q \) and \( H-Q \) relationships based on data when \( S-Q \) or \( H-Q \) were reported together \( (\text{Figure S4}) \). For streams and rivers, both \( S \) and \( H \) exhibit power law
relationships with discharge, \( \log_{10} H = 0.25 \log_{10} Q - 0.46 \) and \( \log_{10} S = -0.12 \log_{10} Q - 2.55 \). For streams and rivers, \( S \) and \( H \) can therefore be predicted by the power laws and corresponding prediction intervals, when at least one of \( S \), \( H \), or \( Q \) were reported. In beaded streams, existing \( S- \) \( Q \) data do not show a clear relationship (Figure S4), and pool structure is expected to behave differently than common streams, so we assumed that ranges of \( H \) and \( S \) for beaded streams do not co-vary with \( Q \).

**SI-3. Non-dimensionalization of the dispersion-reaction equation.**

The original dispersion-reaction equation is

\[
\frac{\partial C(y,t)}{\partial t} = \frac{\partial}{\partial y} \left( D(y) \frac{\partial C(y,t)}{\partial y} \right) - \sum_{\lambda} \phi_{\lambda} aCDOM_{\lambda} (y,t) Q_{dso-\lambda} e^{-K_{d\lambda}(y)t}
\]

(S5)

with non-flux boundary conditions

\[
\frac{\partial C(y,t)}{\partial y} \bigg|_{y=0} = 0
\]

(S6a)

\[
\frac{\partial C(y,t)}{\partial y} \bigg|_{y=H} = 0
\]

(S6b)

where \( C(y,t) \) is DOC concentration, \( t \) is time, \( y \) is the vertical position that is 0 at water surface and positive in the downward direction, and \( D(y) \) is the \( y \)-dependent vertical dispersion coefficient that dictates vertical mixing and sets the rate of resupply of CDOM from bottom waters to the surface, \( \Phi_{\lambda} \) is the photo-lability of CDOM, \( Q_{dso-\lambda} \) is the photon flux at the water surface, \( aCDOM_{\lambda}(y,t) \) is the light attenuation coefficient of CDOM, and \( K_{d\lambda}(y) \) is the total light attenuation coefficient. Notations in this study are summarized in Table S3.

The dimensional variables contained in (S5) are \( C \), \( t \), and \( y \). Thus, the dimensions to be normalized are concentration, time, and length. We define the following scales to normalize dimensional variables into dimensionless ones.

\[
y = H y^*
\]

(S7a)

\[
C(y,t) = \frac{1}{\varepsilon_{280} H} C^* (y^*, t^*)
\]

(S7b)

\[
t = \frac{H^2}{\langle D(y) \rangle} t^*
\]

(S7c)

where superscript * indicates dimensionless quantities, \( \varepsilon_{280} \) denotes empirical parameter \( \varepsilon_{\lambda} \) at wavelength 280 nm, and \( \langle D \rangle \) is the depth-averaged vertical dispersion coefficient. Wavelength 280 nm was picked because it is the high end of the spectra such that \( \varepsilon_{280} \) is always non-zero.

Plug (S7) into (S5)-(S6), (S5)-(S6) become

\[
\frac{\partial C^* (y^*, t^*)}{\partial t^*} = \frac{\partial}{\partial y^*} \left( \frac{D(y^*)}{\langle D \rangle} \frac{\partial C^* (y^*, t^*)}{\partial y^*} \right) - \frac{H^2}{\langle D \rangle} \sum_{\lambda} \phi_{\lambda} aCDOM_{\lambda} (y^*, t^*) \frac{Q_{dso-\lambda}}{C(y^*, t^*)} e^{-K_{d\lambda}(y)t^*} C^* (y^*, t^*)
\]

(S8)

with non-flux boundary conditions
\[
\frac{\partial C^*(y^*,t^*)}{\partial y^*}\bigg|_{y^*=0} = 0 \quad \text{(S9a)}
\]

\[
\frac{\partial C^*(y^*,t^*)}{\partial y^*}\bigg|_{y^*=1} = 0 \quad \text{(S9b)}
\]

(S8) can be rewritten as

\[
\frac{\partial C^*(y^*,t^*)}{\partial t^*} = \frac{\partial}{\partial y^*} \left( D \frac{\partial C^*(y^*,t^*)}{\partial y^*} \right) - \sum \lambda d^*_\lambda e^{-p^*_\lambda y^*} C^*(y^*,t^*) \quad \text{(S10)}
\]

where \( d^*_\lambda \) and \( p^*_\lambda \) are wavelength-specific dimensionless parameters defined by

\[
d^*_\lambda = \frac{H^2}{\langle D \rangle} \frac{aCDOM_{\lambda}(y^*,t^*)}{C(y^*,t^*)} \quad \text{(S11a)}
\]

\[
p^*_\lambda = K_{d,\lambda}(y^*)H \quad \text{(S11b)}
\]

SI-4. Solving for the depth-integrated photomineralization rate.

The generalized dispersion-reaction equation is:

\[
\frac{\partial C^*(y^*,t^*)}{\partial t^*} = \frac{\partial}{\partial y^*} \left( D \frac{\partial C^*(y^*,t^*)}{\partial y^*} \right) - d^* e^{-p^* y^*} C^*(y^*,t^*) \quad \text{(S12)}
\]

with non-flux boundary condition (S9), where \( d^* \) is surface Damkohler number, and \( p^* \) is dimensionless light attenuation. The moment method solves for the dimensionless depth-integrated photomineralization rate, \( r^* \), as well as the vertical distribution of DOC concentration, at asymptotic regime. At the asymptotic regime, the dimensionless depth-averaged DOC concentration decays at a first-order rate constant \( r^* \) over time

\[
\frac{d <C^*>}{dt^*} = r^* <C^*> \quad \text{(S13)}
\]

The moment method defines a representative unit cell, where the vertical water column is mirrored, such that reaction and dispersion profiles are symmetric about \( y^* = 0 \). The governing equation within a unit cell becomes:

\[
\frac{\partial C^*(y^*,t^*)}{\partial t^*} = \frac{\partial}{\partial y^*} \left( D^* \frac{\partial C^*(y^*,t^*)}{\partial y^*} \right) - d^* e^{-p^* |y^*|} C^*(y^*,t^*) \quad \text{(S14)}
\]

with periodic boundary conditions:

\[
C^*(y^*=-1,t^*) = C^*(y^*=1,t^*) \quad \text{(S15a)}
\]

\[
\left. \frac{\partial C^*(y^*,t^*)}{\partial y^*} \right|_{y^*=-1} = \left. \frac{\partial C^*(y^*,t^*)}{\partial y^*} \right|_{y^*=1} \quad \text{(S15b)}
\]

At asymptotic regime, DOC concentration has been fully developed in the unit cell, such that solute concentration is symmetric about \( y^* = 0 \). Therefore, non-flux boundary condition (S9) applies in vertical water column.
The asymptotic regime solution $r^*$ is given by the smallest eigenvalue of the following eigenvalue problem\textsuperscript{10,11}

$$\frac{d}{dy^*} \left( D^*(y^*) \frac{dE(y^*)}{dy^*} \right) - d^* e^{-r^* y^*} |E(y^*)| = -\Lambda E(y^*)$$  \hspace{1cm} (S16)

with periodic boundary conditions:

$$E(y^* = -1) = E(y^* = 1)$$ \hspace{1cm} (S17a)

$$\left. \frac{dE(y^*)}{dy^*} \right|_{y^* = -1} = \left. \frac{dE(y^*)}{dy^*} \right|_{y^* = 1}$$ \hspace{1cm} (S17b)

where $E(y^*)$ is the eigenfunction and $\Lambda$ is the eigenvector. The eigenvector that corresponds to the smallest eigenvalue is the vertical distribution of DOC concentration at asymptotic regime.

We used finite difference method to solve the eigenvalue problem (S16)-(S17). We discretized the vertical water column into $n + 1$ layers ($n = 1024$). The unit cell is therefore discretized into $2n + 2$ layers, each with thickness $h = \frac{1}{n + 1}$. For each layer $i$ ($i = 0, 1, ..., 2n + 1$) within the unit cell, denote $y^*_i \equiv -1 + hi$ such that $y^*_0 = -1$ and $y^*_{n + 1} = 0$.

Denote $D^*_i \equiv D^*(y^*_i)$ and $E_i \equiv E(y^*_i)$, (S12) becomes

$$\frac{D^*_i E_{i+1}}{h^2} + \frac{D^*_{i-1} E_{i-1}}{h^2} - \frac{(D^*_{i+1} + D^*_{i-1}) E_i}{h^2} - d^* e^{-r^* y^*_i} E_i = -\Lambda E_i$$  \hspace{1cm} (S18)

Define

$$\eta_i \equiv -\frac{D^*_i + D^*_{i-1}}{h^2} - d^* e^{-r^* y^*_i}$$ \hspace{1cm} (S19a)

$$\theta_i \equiv \frac{D^*_{i-1}}{h^2}$$ \hspace{1cm} (S19b)

$$\xi_i \equiv -\frac{D^*_{i+1}}{h^2}$$ \hspace{1cm} (S19c)

(S18) becomes

$$\xi_{i-1} E_{i-1} + \eta_i E_i + \theta_{i} E_{i+1} = -\Lambda E_i$$  \hspace{1cm} (S20)

With periodic boundary conditions (S17) applied, (S20) can be written as
To this point, the smallest eigenvalue and the corresponding eigenvector can be solved.


In the Kuparuk River, the vertical dispersion coefficient $D(y)$ follows a standard model\cite{12},\cite{13}:

$$D(y) = \kappa u^* y (1 - \frac{y}{H})$$  \hspace{1cm} (S22)$$

where $\kappa$ is the von Karman constant, $u^*$ is the shear velocity [m s$^{-1}$], and $H$ is the water column depth [m]. Because the observed river channel width is always at least 10 times larger than the observed river depth, turbulent properties are independent of river width\cite{12}. Therefore, we approximate the shear velocity by $u^* = \sqrt{gHS}$, where $g$ is gravitational acceleration [m s$^{-2}$] and $S$ is the water surface slope. The depth-averaged vertical dispersion coefficient therefore is

$$\langle D(y^*) \rangle = \frac{\kappa}{6} u^*H$$  \hspace{1cm} (S23)$$

We assume arctic streams and rivers to follow the same vertical dispersion profile as equation (S22). Arctic lakes and beaded stream pools usually have more complicated dispersion profiles. We assume two scenarios to capture the natural ranges of vertical mixing in beaded stream pools: the vertical dispersion profile in equation (S22) at fast flow, and a layered dispersion profile at slow flow. In the latter case, we assume that the top 10% of the pool depth mixes rapidly at the mean dispersion value of the fast flow scenario, while the deep layer of the pool mixes 2 orders of magnitude slower, a typical ratio for lakes worldwide\cite{14-16}.

$$D(y) = \begin{cases} 
\frac{\kappa}{6} u^*H, & y \leq 10\%H \\
1\% \frac{\kappa}{6} u^*H, & y > 10\%H
\end{cases}$$  \hspace{1cm} (S24)$$

In arctic lakes, because the epilimnion (or the surface layer) usually defines a turbulent mixing layer that has much higher vertical dispersion than the deep layer, we define two scenarios of vertical dispersion: deep lakes that are deeper than the mean epilimnion depth in arctic lakes, and shallow lakes that are shallower than the mean epilimnion depth. Water transparency and mixing depth often relates to each other, because rapid absorption of light at the surface often creates a "trapping depth" defined by a high density gradient\cite{17}. A strong relationship between transparency and mixing depth was found in a variety of lakes, including those larger than Toolik Lake (1.5 km$^2$ fetch) and those that are sheltered from the wind (crater lakes)\cite{18}. Even in sheltered lakes where diurnal thermoclines are more common during solar heating and calm conditions, the longer-term, average mixing depth is still related strongly to the optical properties of the water\cite{18-20}. Secchi depth, which characterizes the optical property of the water column, strongly
affects epilimnion depth in relatively small temperate lakes (defined as fetch < 500 ha in \(^2\)), and
becomes less effective in predicting epilimnion depth of large temperate lakes (defined as fetch >
500 ha in \(^2\)). However, very large lakes in the Arctic are rare compared to temperate lakes, and
studies typically report a much smaller range of fetch (0.003-1.5 km\(^2\), which is 0.3-150 ha, based
on 7 studies of 51 arctic lake\(^2\)). Further, given that arctic waters are typically light limited and
have high CDOM, even if mixing is confined to a near-surface layer, the amount CDOM is
difficult to consume in short periods of time (days to weeks), and CDOM would be resupplied
from greater depths as soon as a diurnal thermocline is erased, for example by wind mixing or
convective mixing at night\(^2\). Therefore, Secchi depth was chosen to estimate epilimnion depth.
We estimate the mean epilimnion depth in arctic lakes using an empirical relationship with
Secchi depth reported for temperate-zone lakes\(^1\), assuming that arctic lakes behave similarly to
temperate-zone lakes
\[ \bar{E}_d = 3.24 + 0.35\bar{S}_d \]  
(S25)
where \( \bar{E}_d \) is the mean epilimnion depth (4.3 m) and \( \bar{S}_d \) is the mean Secchi depth reported in arctic
lakes. In shallow lakes, we treat the entire water column as the surface layer where vertical
dispersion is uniform over depth and spans the range \( 10^{-5} \text{--} 10^{-2} \text{ m}^2 \text{ s}^{-1} \). In deep lakes, we assumed
that deep-layer dispersion is 2 orders of magnitude lower than surface layer dispersion, which is
representative of reported ranges for lakes and oceans\(^14\text{--}16\). For each deep lake, given that Secchi
depth and lake depth are usually not reported together (Table S1), we assumed that the
epilimnion depth (or the surface layer thickness) \( \bar{E}_d \) can be estimated by
\[ E_d = \frac{\bar{E}_d}{\bar{H}_{DL}} H \]  
(S26)
where \( \bar{H}_{DL} \) is the mean depth of deep arctic lakes (12.5 m). (S26) only serves as a rough estimate
of \( \bar{E}_d \) for each deep lake system.
Table S1. Data sources of photochemical and hydrological parameters in arctic stream, rivers, beaded streams, and lakes.

<table>
<thead>
<tr>
<th></th>
<th>DOC</th>
<th>aCDOM$_k$</th>
<th>$\Phi_\lambda$</th>
<th>$Q$</th>
<th>$H$</th>
<th>$S$</th>
<th>SD</th>
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<td>1, 7, 32</td>
<td>1, 7</td>
<td>2, 32-35</td>
<td>2, 35</td>
<td>2, 33, 34</td>
<td>-</td>
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<td>streams</td>
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<td>1, 7, 38</td>
<td>1, 7</td>
<td>2, 33, 35, 36, 39, 40</td>
<td>2, 35</td>
<td>2, 33, 34, 39, 40</td>
<td>-</td>
</tr>
<tr>
<td>beaded streams</td>
<td>1, 29-31, 41-44</td>
<td>38, 41, 43</td>
<td>1, 41, 42</td>
<td>33, 38-40, 43-47</td>
<td>38, 44-47</td>
<td>33, 39, 40, 45, 47</td>
<td>-</td>
</tr>
<tr>
<td>lakes</td>
<td>7, 21, 26, 31, 37, 48-52</td>
<td>1, 7</td>
<td>1, 7</td>
<td>-</td>
<td>21-27, 48, 49, 53</td>
<td>-</td>
<td>31, 54-58</td>
</tr>
</tbody>
</table>

* From left to right, DOC, aCDOM$_k$, $\Phi_\lambda$, $Q$, $H$, $S$, and SD denote DOC concentration, light attenuation coefficient of CDOM, photo-lability of CDOM, flow discharge, water column depth, water surface slope, and Secchi depth respectively. The reference numbers correspond to the references in SI.
Table S2. Reported ranges (min, max) of photochemical and hydrological parameters in arctic streams and rivers (rocky bottom), beaded streams (peat bottom), and lakes.

<table>
<thead>
<tr>
<th></th>
<th>$DOC$ (mol m$^{-3}$)</th>
<th>$aCDOM_{300}$ (m$^{-1}$)</th>
<th>$\Phi_{300}$ (mol CO$_2$ mol$^{-1}$ photon)</th>
<th>$H$ (m)</th>
<th>$S$ (-)</th>
<th>$SD$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>rivers</td>
<td>0.028, 18</td>
<td>3.6, 103</td>
<td>$4\times10^{-4}, 0.08$</td>
<td>0.14, 15.5</td>
<td>5$\times10^{-5}, 0.03$</td>
<td>-</td>
</tr>
<tr>
<td>streams</td>
<td>0.028, 18</td>
<td>4.6, 334</td>
<td>$5\times10^{-4}, 0.04$</td>
<td>0.015, 2.1</td>
<td>1.7$\times10^{-4}, 0.1$</td>
<td>-</td>
</tr>
<tr>
<td>beaded streams</td>
<td>0.12, 3.1</td>
<td>33, 359</td>
<td>0.003, 0.04</td>
<td>0.12, 3.0</td>
<td>3$\times10^{-4}, 0.009$</td>
<td>-</td>
</tr>
<tr>
<td>lakes</td>
<td>0.008, 2.7</td>
<td>4.3, 79</td>
<td>$4\times10^{-4}, 0.08$</td>
<td>0.08, 43</td>
<td>-</td>
<td>0.3, 11.2</td>
</tr>
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</table>

* Nomenclatures same as Table S1. Light attenuation coefficient of CDOM ($aCDOM_{\lambda}$) and photo-lability of CDOM ($\Phi_{\lambda}$) are adjusted to wavelength = 300 nm.
Table S3. (A) Percentage of systems partially or substantially limited by mixing for each type of arctic systems that exhibit exponential light attenuation over depth. (B) Depth-integrated attenuation ratio (mean ± standard error) for each type of arctic systems that are partially or substantially limited by mixing.

<table>
<thead>
<tr>
<th></th>
<th>Partially Limited</th>
<th>Substantially Limited</th>
</tr>
</thead>
<tbody>
<tr>
<td>Streams and rivers</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>Beaded streams</td>
<td>0.1%</td>
<td>0%</td>
</tr>
<tr>
<td>Lakes</td>
<td>12%</td>
<td>30%</td>
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<table>
<thead>
<tr>
<th></th>
<th>Partially Limited</th>
<th>Substantially Limited</th>
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<tr>
<td>Streams and rivers</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Beaded streams</td>
<td>0.92±0.001</td>
<td>---</td>
</tr>
<tr>
<td>Lakes</td>
<td>0.95±0.001</td>
<td>0.58±0.002</td>
</tr>
</tbody>
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**Table S4. List of notations.**

### English letters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_{CDOM}$</td>
<td>light attenuation coefficient of CDOM</td>
</tr>
<tr>
<td>$a_{SS}$</td>
<td>light attenuation coefficient of suspended sediment (SS)</td>
</tr>
<tr>
<td>$C$</td>
<td>DOC concentration</td>
</tr>
<tr>
<td>$D$</td>
<td>vertical dispersion coefficient</td>
</tr>
<tr>
<td>$d^*$</td>
<td>surface Damkohler number</td>
</tr>
<tr>
<td>$E$</td>
<td>eigenfunction</td>
</tr>
<tr>
<td>$E_d$</td>
<td>epilimnion depth</td>
</tr>
<tr>
<td>$g$</td>
<td>gravitational acceleration</td>
</tr>
<tr>
<td>$H$</td>
<td>water column depth</td>
</tr>
<tr>
<td>$H_{DL}$</td>
<td>mean depth of deep arctic lakes</td>
</tr>
<tr>
<td>$K_{d\lambda}$</td>
<td>total light attenuation coefficient</td>
</tr>
<tr>
<td>$p^*$</td>
<td>dimensionless light attenuation</td>
</tr>
<tr>
<td>$P_{e}$</td>
<td>statistical equilibrium of the vertical distribution of DOC</td>
</tr>
<tr>
<td>$PM$</td>
<td>DOC photo-mineralization rate</td>
</tr>
<tr>
<td>$Q$</td>
<td>discharge</td>
</tr>
<tr>
<td>$Q_{dso-\lambda}$</td>
<td>photon flux at the water surface</td>
</tr>
<tr>
<td>$Q_{\lambda}$</td>
<td>photon flux</td>
</tr>
<tr>
<td>$r$</td>
<td>depth-integrated photomineralization rate</td>
</tr>
<tr>
<td>$r_{wm}$</td>
<td>depth-integrated photomineralization rate under well-mixed assumption</td>
</tr>
<tr>
<td>$S$</td>
<td>water surface slope</td>
</tr>
<tr>
<td>$S_d$</td>
<td>Secchi depth</td>
</tr>
<tr>
<td>$t$</td>
<td>time</td>
</tr>
<tr>
<td>$u^*$</td>
<td>shear velocity</td>
</tr>
<tr>
<td>$W$</td>
<td>river width</td>
</tr>
<tr>
<td>$y$</td>
<td>vertical position in the water column</td>
</tr>
</tbody>
</table>

### Greek letters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_{\lambda}$</td>
<td>reaction efficiency</td>
</tr>
<tr>
<td>$\varepsilon_{\lambda}$</td>
<td>von Karman constant</td>
</tr>
<tr>
<td>$A$</td>
<td>eigenvector</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>wavelength</td>
</tr>
<tr>
<td>$\phi_{\lambda}$</td>
<td>photo-lability of CDOM to photomineralization, also known as the apparent quantum yield for photomineralization</td>
</tr>
</tbody>
</table>

### Others

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>superscript $^*$</td>
<td>dimensionless variable or parameter</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>-------</td>
<td>----------------------</td>
</tr>
<tr>
<td>\bar{}</td>
<td>depth-averaged value</td>
</tr>
<tr>
<td>\mu</td>
<td>mean value</td>
</tr>
</tbody>
</table>
References


43. B. L. Miller, Terrestrial-aquatic transfers of carbon dioxide, methane, and organic carbon from riparian wetlands to an arctic headwater stream, M.S., University of Michigan at Ann Arbor, 2014.


47. A. Tarbeeva and V. Surkov, Beaded channels of small rivers in permafrost zones, *Geography and Natural Resources*, 2013, 34, 216-220.


