Capillary Descent

Joachim Delannoy, Hélène de Maleprade, Christophe Clanet & David Quéré

Physique & Mécanique des Milieux Hétérogènes, UMR 7636 du CNRS, ESPCI, 75005 Paris, France.
LadHyX, UMR 7646 du CNRS, École polytechnique, 91128 Palaiseau, France.

Supplementary Information

Aerophilic coating

Figure SI1. (a) Topography of a glass surface treated with Glaco and imaged by atomic force microscopy. Courtesy: Philippe Bourrianne. (b) Scanning electron microscopy images of a capillary tube ($r = 0.51$ mm) coated with Glaco. The coating layer is much thinner than the tube radius.

Tilt angle effects in the linear regime of descent

We did experiments with capillary tubes tilted by an angle $\alpha$ to the vertical, which modifies the acceleration of gravity to $g\cos \alpha$. This trick allows us to extend the regime of constant velocity and thus to perform more accurate measurements in the figure 3 of the accompanying paper. We discuss here how the tilt impacts the speed of descent. Two tubes with different dimensions are tested, and the velocity of the meniscus is reported as a function of $\alpha$ in figure SI-2. We observe that the velocity varies from less than 10% while the gravity is changed by a factor 2, which confirms the independence of $V$ towards $\alpha$ predicted by equation 3.
Figure SI2. Effects of the tilt angle $\alpha$ of tubes (radius $r$, length $L$) on the capillary descent velocity $V$. Dots show experimental results whose average value is indicated with dashed lines. These averaged values are the ones reported in the figure 3 of the accompanying paper.

**Relaxation time**

The relaxation time of the meniscus as it reaches its equilibrium depth is obtained by balancing the viscous friction $F_\eta = -8\eta r(L-z)\dot{z}$ with gravity $F = -\pi r^2 \rho g (z-H)$. The solution of this equation is $\ln (1-z/H) - (z-H)/(L-H) = C - t/\tau$ where $C$ is a constant and $\tau = 8\eta(L-H)/r\rho g r^2$ is the characteristic time of relaxation.

The relaxation time is experimentally obtained by fitting the curves around the equilibrium position (see figure SI3.a). Data in this semi-logarithmic representation indeed align along straight lines, in agreement with the expected exponential relaxation. We deduce from the fits (thin straight lines) the characteristic time $\tau$ of relaxation, which we plot it in figure SI3.b as a function of the quantity $(L-H)/(r^2 \cos \alpha)$, where the set of data was augmented by performing experiments with tubes inclined by an angle $\alpha$ toward the vertical. All data collapse on a line whose slope ($7 \times 10^{-7}$ m.s) nicely compares to $8\eta/\rho g \approx 8 \times 10^{-7}$ m.s, the value expected from the model.
Figure SI3. (a) Relaxation stage as a meniscus (position $z$) inside a superhydrophobic tube (length $L$) immersed in water relaxes to its equilibrium depth $H$. The lines are linear fits, from which we deduce for each curve the characteristic time $\tau$ of the relaxation. (b) Relaxation time $\tau$ for menisci in tubes of length $L$ and radius $r$ and tilted to the vertical by an angle $\alpha$, and plotted as a function of $(L-H)/r^2$cos $\alpha$; the line shows a linear fit of slope $7 \times 10^{-7}$ m.s.

Effects of pressure loss at the exit/entrance of the tube

Following the analysis developed in É. Lorenceau et al., Phys. Fluids, 14, 1985–1992 (2002), an inertial additional term $F_c = -2\pi r^2 \rho z^2$ has to be accounted for when the liquid rises, corresponding to the singular pressure loss associated to a sharp constriction in a pipe. As seen in the figure SI4, this extra term (red dashed line) provides an accurate fit to the data (black dots), significantly better than without this correction (black dashed line).

Figure SI4. Evolution of the meniscus position for a capillary tube of length $L = 7.5$ cm and radius $r = 1.5$ mm. Dots show experimental measurements and dashed lines the numerical integration of the model without (black line) and with (red line) the pressure loss at the tube entrance.