Supplementary material to
Effect of microchannel structure and fluid properties on non-inertial particle migration


Figure 1: Instantaneous image of experiments in an orderly arranged pillared microchannel of porosity, $\epsilon = 0.76$ (a) Boger fluid (F2) at $Wi = 3.3$ (b) HPAM3630 (F4) at $Wi = 1391$. 
Figure 2: Experimental results for particle fraction in an orderly arranged pillared microchannel of porosity, $\epsilon = 0.76$ (a) Boger fluid (F2) (b) HPAM3630 (F4).

Figure 3: Instantaneous image of experiments in a randomly arranged pillared microchannel of porosity, $\epsilon = 0.81$ in Boger fluid (F2) at Wi = 2.7

**Dimensionless number influencing particle migration**

Using Oldroyd-B constitutive equation for evaluating first normal stress difference, $N_1$ ([1]):

$$N_1 \sim \eta \lambda \dot{\gamma}^2$$

(1)
Figure 4: Distribution of (a) velocity and (b) shear rate along the width for Newtonian fluid (F1), Boger fluid (F2) and HPAM3630 (F4) at various \( Wi \).

Lateral force on the particle can be estimated as:

\[
F_y \sim \frac{\pi d_p^3}{6} \frac{\partial N_1}{\partial y} \tag{2}
\]

Lateral velocity of a particle based on Stokes flow assumption:

\[
u_y = \frac{F_y}{3\pi \eta d_p} \sim d_p^2 \lambda \dot{\gamma} \frac{\partial \dot{\gamma}}{\partial y} \tag{3}
\]

Assuming parabolic velocity profile,

\[
u_x = u \left( 1 - \left( \frac{2y}{w} \right)^2 \right) \tag{4}
\]

\[
\dot{\gamma} = \frac{\partial u_x}{\partial y} = \frac{4u_y y}{w^2} \tag{5}
\]

\[
\frac{\partial \dot{\gamma}}{\partial y} = \frac{4u}{w^2} \tag{6}
\]

\[
u_y \sim \frac{\lambda u^2 d_p^2 y}{w^4} \tag{7}
\]

\[
\frac{dy}{dx} = \frac{u_y}{u_x} \sim \frac{\lambda u d_p^2 y}{w^4 \left( 1 - \left( \frac{2y}{w} \right)^2 \right)} \tag{8}
\]

Non-dimensionalizing the \( y \) distance with half-width \((w/2)\) and \( x \) distance with the length of a channel \( l \): \( \tilde{y} = 2y/w, \; \tilde{x} = x/l \)

\[
\frac{\tilde{y}}{\tilde{x}} \sim \frac{l}{w} \frac{\lambda u d_p^2 \tilde{y}}{w^3 \left( 1 - \tilde{y}^2 \right)} \tag{9}
\]

Relevant dimensionless group for particle migration is:
\[
\frac{\lambda ud_p l}{w^4} = \frac{\lambda u}{w} \frac{l}{w} \left( \frac{d_p}{w} \right)^2 = Wi(l/w) \beta^2
\]  

(10)

Similar dimensionless group is also proposed by [2,3] and is valid in the limit of small Weissenberg numbers \(Wi \ll 1\).

References

