 Electrostatic intermolecular interactions are treated in the point dipole approximation, so that

\[ J_{i,j} = \frac{1}{4\pi \varepsilon} \left( \vec{\mu}_i \cdot \vec{\mu}_j - 3(\vec{\mu}_i \cdot \vec{d}_{ij})(\vec{\mu}_j \cdot \vec{d}_{ij}) \right) \]

\[ = \frac{\mu_0^2}{4\pi \varepsilon d_{ij}^3} \left[ \vec{m}_i \cdot \vec{m}_j - 3(\vec{m}_i \cdot \vec{d}_{ij})(\vec{m}_j \cdot \vec{d}_{ij}) \right], \]  

where \( \vec{\mu}_i \) is the transition dipole moment on molecule \( i \), \( \vec{m}_i = \vec{\mu}_i / \mu_0 \) is the unit vector that defines the direction of \( \vec{\mu}_i \), \( \vec{d}_{ij} \) is the unit vector connecting \( i \) and \( j \) sites, whereas \( d_{ij} \) is the distance between the two dipoles. The term in the squared parenthesis in the right hand side of the second line of the above equation is the \( D_{ij} \) quantity in Eq. 2 in the main text.

To relate the total oscillator strength of a system to a ground-state expectation value, we define the velocity dipole operator, \( \hat{v} \):

\[ \hat{v} = [\hat{\mu}, \hat{H}], \]  

where, to simplify the notation, we have suppressed the vector notation on both \( \hat{\mu} \) and \( \hat{H} \) operators. The oscillator strength associated with the \( G \rightarrow E \) transition is:

\[ f_{EG} = \frac{2}{3} \frac{m_e}{\hbar c^2} \omega_{EG} \langle G|\hat{\mu}|E\rangle\langle E|\hat{\mu}|G\rangle. \]  

To eliminate the transition frequency from the above expression we use:

\[ i\langle G|\hat{v}|E\rangle = \omega_{EG}\langle G|\hat{\mu}|E\rangle \]

and its complex conjugate, thus getting:

\[ F = \sum_E f_{EG} = -\frac{im_e}{3\hbar c^2} \langle G|\hat{\mu}, \hat{v}|G\rangle, \]

thus proving Eq.12 in the main text. Furthermore, using Eqs. 3 and 9 (main text) and remembering the Paulions algebra (see Eq. 4 in the main text), the commutator \([\hat{\mu}, \hat{v}]\) can be easily calculated. In particular, we have:

\[ [\hat{\mu}, \hat{v}] = \frac{1}{i\hbar} [\hat{\mu}, [\hat{\mu}, \hat{H}]], \]

\[ = \frac{1}{i\hbar} \mu_0^2 \sum_i [\hbar \omega_0 - \lambda (\hat{a}_i^\dagger + \hat{a}_i)](4\hat{\mu}_i - 2), \]  

that is Eq. 14 in the main text.
Figure 1: The same results as in Fig. 2, main text, but accounting for long-range electrostatic interactions.
Figure 2: The same results as in Fig. 3, main text, but accounting for long-range electrostatic interactions.

Figure 3: The same results as in Fig. 2, main text, but for different electron-vibration coupling strength. a) $J_1 = 0.255 \text{ eV}, \lambda = 0.10 \text{ eV}$; b) $J_1 = 0.255 \text{ eV}, \lambda = 0.22 \text{ eV}$; c) $J_1 = -0.255 \text{ eV}, \lambda = 0.10 \text{ eV}$; d) $J_1 = -0.255 \text{ eV}, \lambda = 0.22 \text{ eV}$. 

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Figure 4: The same results as in Fig. 3, main text, but for different electron-vibration coupling strength. a) $J_1 = 0.255 \text{ eV}$, $\lambda = 0.10 \text{ eV}$; b) $J_1 = 0.255 \text{ eV}$, $\lambda = 0.22 \text{ eV}$; c) $J_1 = -0.255 \text{ eV}$, $\lambda = 0.10 \text{ eV}$; d) $J_1 = -0.255 \text{ eV}$, $\lambda = 0.22 \text{ eV}$. Please notice the different emission intensity scales on the y axis.

Figure 5: The same results as in Fig. 7, main text, but accounting for long-range electrostatic interactions. The number of molecules included in the calculation has been set to 6, the biggest aggregate that can be computed with a 3PA-lr basis.
Figure 6: The same results as in Fig. 8, main text, but accounting for long-range electrostatic interactions. The number of molecules included in the calculation has been set to 6, the biggest aggregate that can be computed with a 3PA-lr basis. For H aggregates the monomer emission intensity is rescaled by a factor of $1/50, 1/100$ and $1/1000$ in panels (b), (d) and (f), respectively.