Flow stabilization in wearable microfluidic sensors enables noise suppression

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Supplementary information

1. The calculation of membrane capacitance:

The peak deflection of a circular membrane with a thickness, \( t \), is given by:

\[
d_{pk} = \frac{12Pa^4(1 - \nu^2)}{64Et^3}
\]

Eq.1

where, \( P \) is the applied pressure, \( a \) is the radius, \( E \) is the Young’s modulus. For elastomeric membranes, Poisson ratio, \( \nu \), approaches 0.5. Therefore, \((1 - \nu^2)\) in the numerator can be written as 0.75.

The average deflection along the radius of this circle is calculated as:

\[
d_{avg} = \int_0^a \frac{d(r)}{a} = \frac{d_{pk}}{5} = \frac{9Pa^4}{320Et^3}
\]

Eq.2

If we assume that the deflection for a long rectangular channel membrane with length, \( L \), is the same as the circular membrane deflection along its radius, then the volume change in a rectangular channel can be written as:

\[
\Delta V = d_{avg}Lw
\]

Eq.3

where, \( w \), is the channel width and is equivalent to 2\( a \). Finally, by inserting the Eq.2 in Eq.3, the membrane capacitance can be written as;
\[
C = \frac{dV}{dP} = \frac{w^5 L}{569 E t^3}
\]

Eq. 4

2. The derivation of approximate time constant equation:

We first assume an aspect ratio of two, giving us a channel height of 2w for a channel width of w for both the LR and the SC. In this case, assuming w is smaller than membrane thickness, t, \(C_M\) is negligible in comparison to \(C_W\). If the number of concentric rings, \(n\) is much larger than one, then we can neglect the resistance of the LR in comparison to SC and neglect the capacitance of SC in comparison to LR. Assuming there is no inlet capacitance and for an ideal elastomer (\(\nu=0.5\));

\[
C_{total} = C_M = \frac{3nw^2L}{E}
\]

Eq. 5

\[
R_{total} = \frac{12\eta L}{2w^4(1 - 0.63 \frac{w}{2w})} = \frac{8.76\eta L_{sc}}{w^4}
\]

Eq. 6

Multiplying \(R_{total}\) and \(C_{total}\) in Eq. 5 and 6, gives us the time constant;

\[
\tau = \frac{26.3n\eta LL_{sc}}{Ew^2}
\]

Eq. 7

where, \(L\) is the circumference of liquid reservoir and \(L_{sc}\) is the filled sensing channel length.