Supplementary Material: Poynting effect of brain matter in torsion

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1 Testing protocol validation

To validate the testing protocol, we further performed torsion tests on two silicon gel samples (G_1 and G_2) at $12.5\,\mathrm{rad\,m^{-1}}$ twist rate. The radius and the height of the samples prior to twisting are reported in Table 1a.

sample	Gel type	λ	l_0 [mm]	r_0 [mm]
G_1	DragonSkin10	0.998	12.14	4.00
G_2	DragonSkinFXPro	0.998	12.40	4.00

(a) Geometry of the silicon samples after pre-compression, prior to twisting: the estimated axial stretch λ, the length $l_0 = \lambda L_0$ (measured by the instrument), the radius $r_0 = \lambda^{-1/2} R_0$ of the two samples.

sample	μ [Pa]	c_2 [Pa]
G_1	286838.72	20051.70
G_2	191999.03	1954.44

(b) Estimated elastic parameters: the shear modulus $\mu = 2(c_1 + c_2)$, the Mooney-Rivlin parameter c_2.

Table 1: Results of the torsion tests on silicon gel samples.

The results are plotted in Figure 1: the torque and the normal force data (points) are fitted with a Mooney Rivlin model (solid lines) and the coefficient of determination R^2 is shown for each set of data. The estimated elastic parameters μ and c_2 are reported in Table 1b. The normal force data clearly show that a Positive Poynting effect is measured for silicon cylinders as well. (The sign has been inverted from the original output to be consistent with the modelling convention and with the main paper).

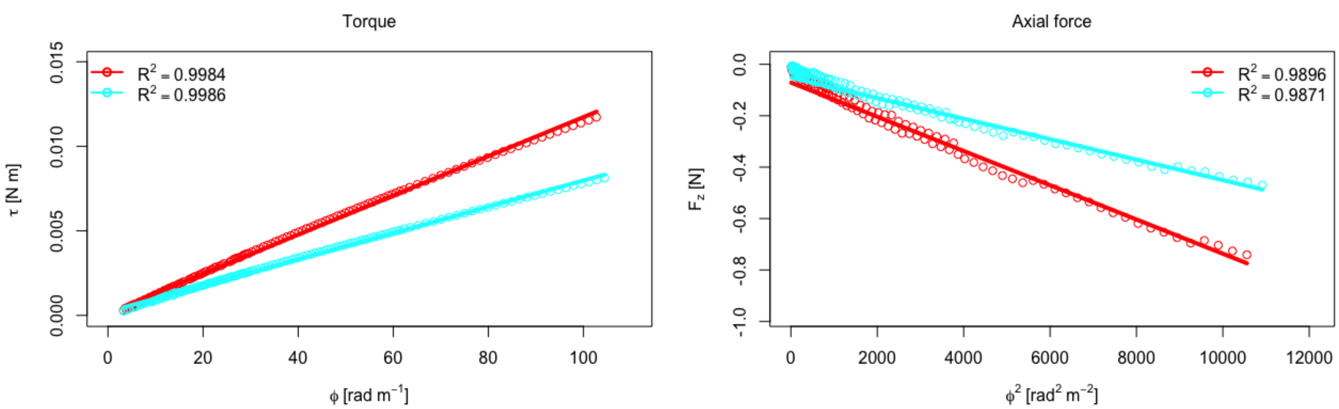

Figure 1: Torque and normal force data collected from the torsion tests at $12.5\,\mathrm{rad\,m^{-1}}$ for sample G_1 (red) and G_2 (cyan); and Mooney-Rivlin model prediction (solid lines).

2 Ramp time

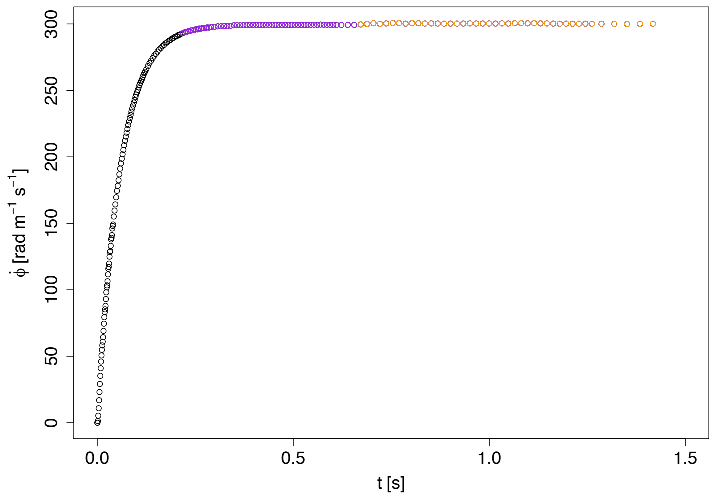

Figure 2: Original collected data on Sample S4: twist rate against time, the ramp data are shown in black; the ramp time is approximately $0.25\,\mathrm{sec}$.

3 Rotational head impact simulations

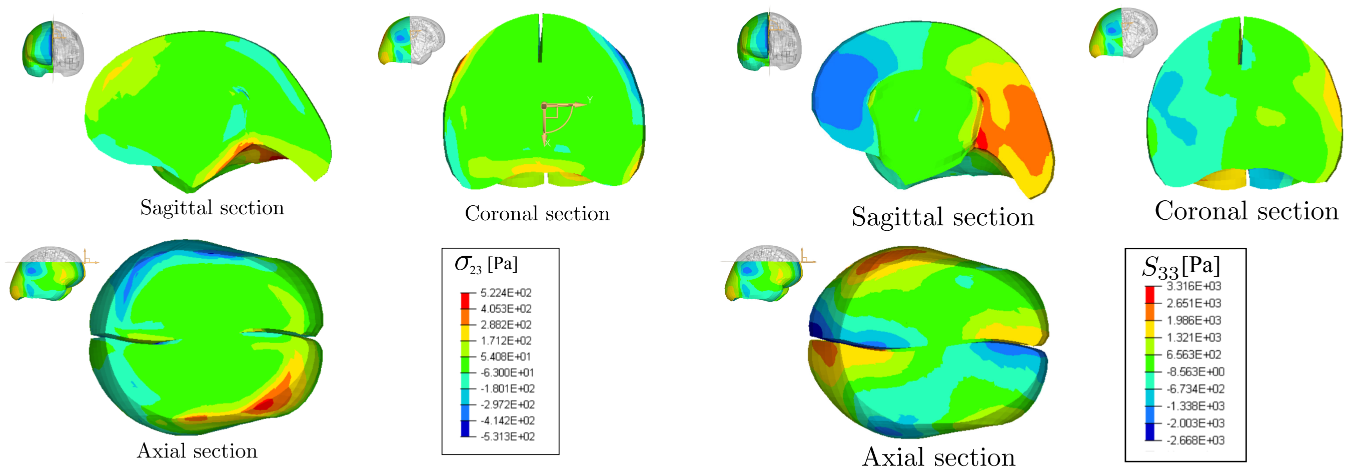

Figure 3: Results of the FE simulations of a rotational head impact, performed with the UCDBT Model. On the left: Distribution of the Cauchy shear stress component σ_{23} across the Sagittal, Coronal and Axial planes; here the orange and blue areas correspond to peak stress magnitude of approximatively 0.4 kPa. On the right: Distribution of the deviatoric vertical stress component S_{33} across the Sagittal, Coronal and Axial planes; here the orange and blue areas correspond to peak stress magnitude of approximatively 2 kPa (the deviatoric stress is the Cauchy stress minus the hydrostatic stress $\mathbf{S} = \boldsymbol{\sigma} - (1/3)\mathrm{tr}[\boldsymbol{\sigma}]\mathbf{I}$.)

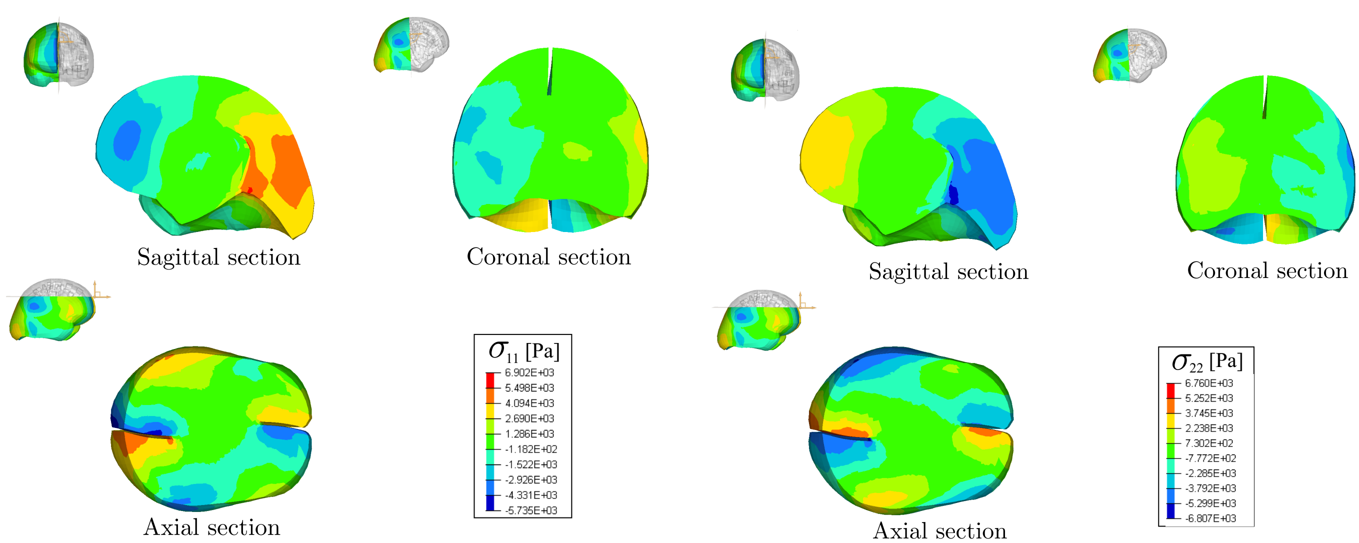

Figure 4: Results of the FE simulations of a rotational head impact, performed with the UCDBT Model. Distribution of the principal Cauchy stress components σ_{11} (on the left) and σ_{22} (on the right) in the Sagittal, Coronal and Axial planes. Here the orange and blue areas correspond to peak stress magnitude of approximatively 4 kPa, same as for the vertical stress component σ_{33}, see Figure 6 in the main paper.

4 A remark on differences and similarities between simple shear and torsion

We briefly compare the results obtained here for torsion tests with those obtained elsewhere for simple shear tests and for torsion modelled as simple shear.

We begin by recalling that the deformation gradient for uni-axial compression in the Z direction, followed by simple shear of amount κ in the YZ plane, has the form[?]

$$\mathbf{F} = \begin{pmatrix} 1/\sqrt{\lambda} & 0 & 0 \\ 0 & 1/\sqrt{\lambda} & \lambda\kappa \\ 0 & 0 & \lambda \end{pmatrix}. \tag{1}$$

Hence, we see from comparison with Equation (2) in the main article that there is a formal connection between torsion and simple shear. However, the equivalence is *local* only, as simple shear is homogeneous but torsion is not: the amount of "shear" experienced by an element in torsion ($\kappa = r\phi = r\alpha/H$) depends on the dimensions of the sample and the position of the element. Thus, it does not make sense to compare the amount of shear and the shear rate experienced by all elements in a simple shear experiment with the amount of "shear" and the "shear" rate experienced by a given element at a given location for a given sample dimension in a torsion experiment. Despite this disconnect, finite shear and torsion are often confused in the literature, and torsion experiments in rheometers are routinely modelled as simple shear, see for example the papers cited in the extensive review by Chatelin et al.[?] .