Figure S1. A. The divergence of the minimum, Newtonian viscosity before shear thickening, as a function of packing fraction. Open circles indicate data that was excluded from fitting due to the presence of shear thinning which may obscure the baseline, Newtonian suspension viscosity. Dashed lines indicate the fit determined by Equation 1, using $\beta = 2$ for all aspect ratio systems. The fitted value of $\phi_0$ is shown by the vertical lines: $\phi_0(AR1) = 0.557$, $\phi_0(AR8) = 0.506$, and $\phi_0(AR11) = 0.452$. B. The divergence of the maximum viscosity at the peak of shear thickening, as a function of packing fraction. Open circles indicate data that was excluded from fitting due to the possibility of the suspension being driven into the shear jammed state during rheology, resulting in an inaccurate measured viscosity. Dotted lines indicate the fit determined by Equation 1, using $\beta = 1.8$ for all aspect ratio systems. The fitted value of $\phi_m$ is shown by the vertical lines: $\phi_m(AR1) = 0.488$, $\phi_m(AR8) = 0.386$, and $\phi_m(AR11) = 0.328$. 
Generation of the State Diagram from the Wyart-Cates Model

The Wyart-Cates (WC) model\(^1\) predicts a suspension with packing fraction \(\phi < \phi_m\) will exhibit two Newtonian flow branches: a low-shear branch with viscosity dependent on the frictionless packing fraction \(\phi_0\), and a high-shear branch with viscosity dependent on the frictional packing fraction \(\phi_m\). Both these viscosities can be described using a Krieger-Dougherty relation\(^2\):

\[
\eta = \eta_0 \left(1 - \frac{\phi}{\phi_J}\right)^\beta
\]

Where \(\eta\) is the suspension viscosity, \(\eta_0\) is the suspending solvent viscosity, \(\phi_J\) is the relevant packing fraction (\(\phi_0\) at low shear and \(\phi_m\) at high shear), and \(\beta\) is a fitting parameter.

The WC model posits that the system transitions between the low-shear and high-shear viscosities in accordance with an effective jamming point, \(\phi_{\text{eff}}\). \(\phi_{\text{eff}}\) is a weighted average of \(\phi_0\) and \(\phi_m\), using the fraction of particles in frictional contact, \(f(\tau) = 1 - e^{-\tau/\tau^*}\), where \(\tau^*\) describes the breakdown of lubrication and the initiation of friction-dominated flows.

\(\phi_{\text{eff}}\) is thus given by

\[
\phi_{\text{eff}} = \phi_m \left(1 - e^{-\tau/\tau^*}\right) + \phi_0 \left(e^{-\tau/\tau^*}\right).
\]

By substituting \(\phi_{\text{eff}}\) into the Krieger-Dougherty description, we find:

\[
\eta = \eta_0 \left(1 - \frac{\phi}{\phi_m + e^{-\tau/\tau^*}(\phi_0 - \phi_m)}\right)^\beta.
\]

Thus, armed with the previously-determined values of \(\phi_0\) and \(\phi_J\), we can fit the experimental flow curve, \(\eta(\tau)\), to this equation and find \(\tau^*\). In all cases, \(\beta\) is a free parameter.

Armed with \(\phi_0, \phi_m, \tau^*,\) and \(\beta\), it is possible to calculate the DST and SJ boundaries and generate a state diagram. This is most easily done as a function of shear stress and shear rate; thus we reformulate the above expression for \(\eta(\phi_{\text{eff}})\):

\[
\dot{\gamma} = \frac{\tau}{\eta_0} \left(1 + \frac{\phi}{\phi_m + (\phi_0 - \phi_m)e^{-\tau/\tau^*}}\right)^\beta.
\]

The SJ boundary can be found by solving this equation for \(\dot{\gamma} = 0\), which will generate two families of answers, including one at \(\tau = 0\) which is the trivial solution at rest. The relevant solution is for nonzero \(\tau\).

The DST boundary occurs when there is a discontinuous increase in the viscosity or shear stress at a given shear rate. Thus, the DST condition is that \(\frac{d\log(\eta)}{d\log(\dot{\gamma})} = \frac{d\log(\tau)}{d\log(\dot{\gamma})} = \infty\). Thus can be inverted to give \(\frac{d\log(\dot{\gamma})}{d\log(\tau)} = 0\), which can be more easily solved. This equation yields two solutions, which form the edges

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of the “nose” of the DST regime.

When these boundaries are plotted as $\tau(\phi)$, they yield the DST and SJ state diagram regions shown in Figure 4. Above $\phi = \phi_0$, the system exhibits isotropic jamming indicated in gray.