Supporting Information

Calculation details:

According to thermal spike theory, the temperature distribution in cylindrical coordinates related to a bombarded spot is: \(^{1,2}\)

\[
T(F_d, r, t, T_s) = \frac{F_d}{4\pi\kappa t}\exp(-\rho C r^2 / 4\kappa t) + T_s \quad (S1)
\]

where \(F_d\) is the energy deposited into the track per unit length, \(\kappa\) is the thermal conductivity of TiO\(_2\) (11.7 W/m/k), \(\rho\) is the density of TiO\(_2\) (3.8 g/cm\(^3\)), \(C\) is the heat capacity of TiO\(_2\) (57.12 J/mol/K), \(t\) is the relaxation time, \(r\) is the related distance to the bombarded spot, and \(T_s\) is the substrate temperature.\(^3\)

As a result, the viscosity on the substrate surface decreases due to thermal activation:

\[
\eta_{\text{thermal}} = \eta_0 \exp(E_{\text{flow}} / kT) \quad (S2)
\]

where \(\eta_0\) is a material dependent prefactor, \(E_{\text{flow}}\) is the activation energy for flow, and \(k\) is Boltzmann’s constant. We can use the first order estimates for \(\eta_0\) and \(E_{\text{flow}}\): \(\eta_0 = 10^{-3}\) Pa·s and \(E_{\text{flow}} = 1.0\) eV. Subsequently, the shear stress can relax in the locally heated region at a rate of \(R = \mu / \eta_{\text{thermal}}\) (where \(\mu\) is shear modulus at a value of 90 GPa),\(^3\) and the temperature is cooled back to the substrate temperature. The total amount of stress relaxation up to a time \(\tau\) is:

\[
\Omega(F_d, r, \tau, T_s) = 1 - \exp\left[\int_0^\tau -R(F_d, r, t, T_s) dt\right] \quad (S3)
\]

The ion-induced viscosity \(\eta\) can be calculated by the cross section \(\theta_{\text{eff}}(F_d, T_s)\) of full stress relaxation:

\[
\eta_{\text{rad}} = \frac{\mu}{3B\theta_{\text{eff}}(F_d, T_s)} = \frac{\mu}{3B \int_0^{2\pi\infty} \Omega(F_d, r, \infty, T_s) r dr d\theta} \quad (S4)
\]

\[
\eta = \frac{\eta_{\text{rad}}}{f} \quad (S5)
\]

where \(B\) is dependent on Poisson’s ratio \(\nu\) by: \(B = 6(1-\nu)/(5-4\nu)\), \(f\) is the ion flux. Poisson’s ratio \(\nu\) for TiO\(_2\) is 0.28.\(^3\)

As shown in Fig. S1, for 30 kV FIB sculpting, the total amount of stress relaxation at 1 ps overlaps with those of 10 ps and when \(\tau \to \infty\), which means the stress has almost
completely relaxed at a time scale of picosecond. Similarly, the time scale of stress relaxation for 2 kV-16 kV FIB sculpting is also picosecond. Monte Carlo simulation by SRIM shows $F_d$ is 1.89 kV/nm, 1.6 kV/nm, 1.27 kV/nm, 1.06 kV/nm, and 0.71 kV/nm for 30 kV, 16 kV, 8 kV, 5 kV, and 2 kV Ga$^+$ beam sculpting, respectively. Therefore, the temperature distribution at local thermal spikes under 2 kV-30 kV FIB sculpting can be calculation by equation (S1) as shown in Fig. 5.

![Graph showing stress relaxation](image)

Fig. S1. Calculations of the total amount of stress relaxation at $T_S=300$ K up to time $\tau$: 0.1 ps, 1 ps, 10 ps, and $\infty$.

References: