

Supplementary Information

Competition between tubular, planar and cage geometries: a complete picture of structural evolution of B_n ($n = 31\text{--}50$) clusters

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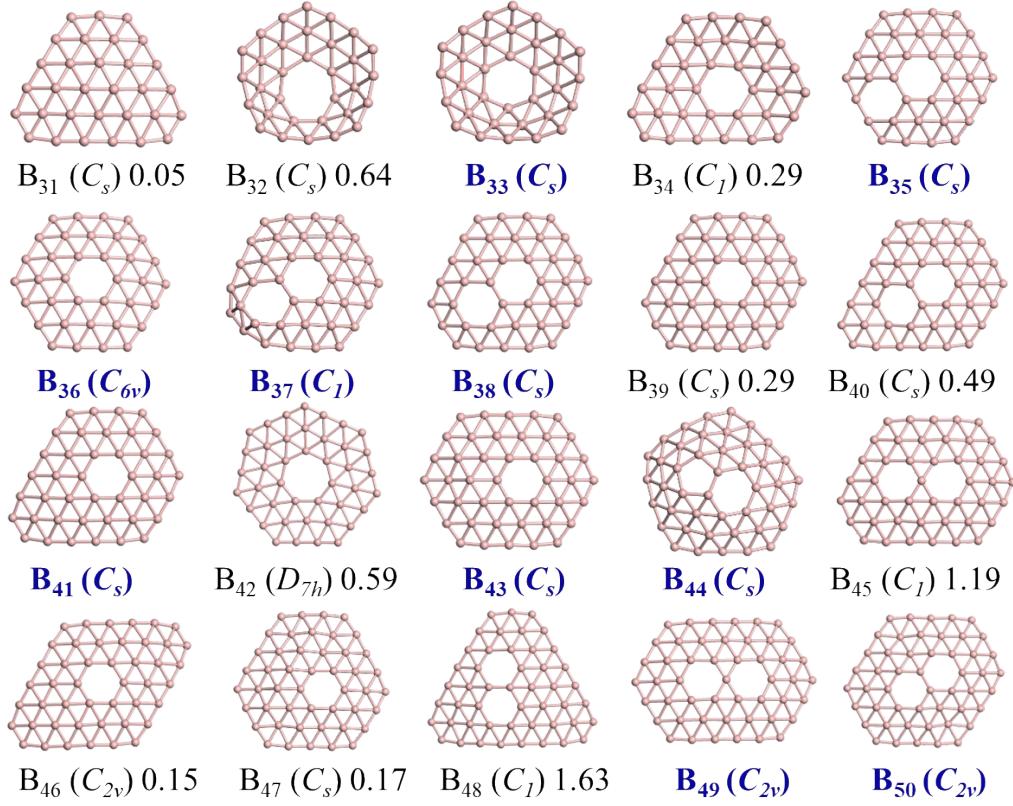


Fig. S1 The quasi-planar structures of neutral B_n ($n = 31\text{--}50$) clusters from TPSSh/6-311+G* calculations. The cluster symmetry and relative energy (in eV) are given at the below each structure. The lowest energy structures are marked in blue.

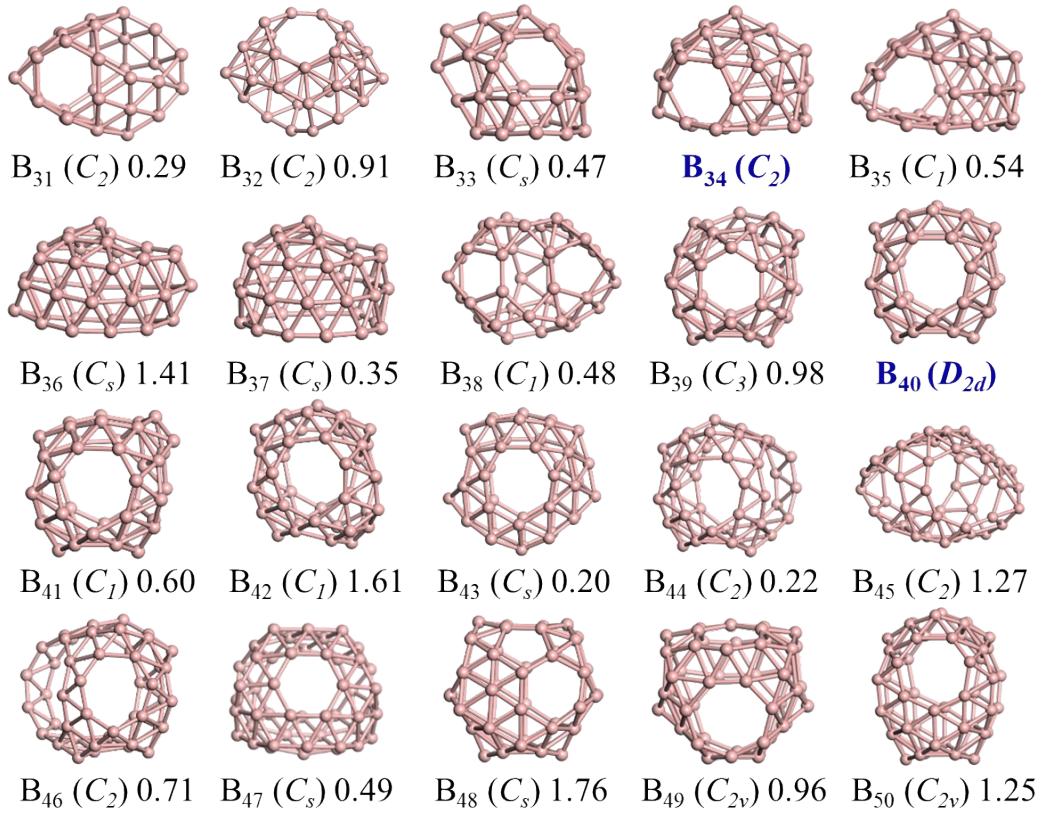


Fig. S2 The cage-like structures of neutral B_n ($n = 31\text{--}50$) clusters from TPSSh/6-311+G* calculations. The cluster symmetry and relative energy (in eV) are given at the below of each structure. The lowest energy structures are marked in blue.

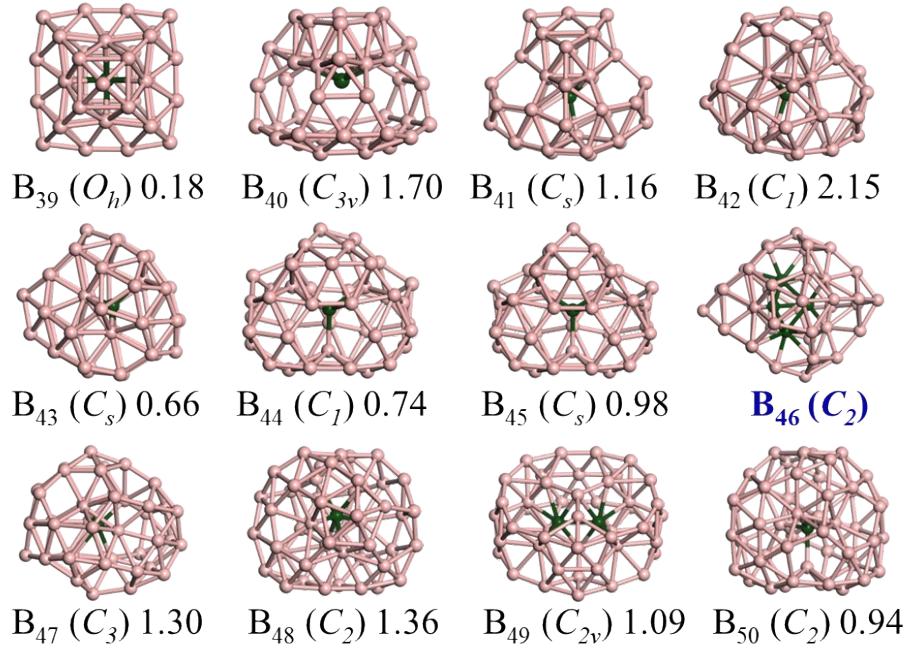


Fig. S3 The core-shell structures of neutral B_n ($n = 39\text{--}50$) clusters from TPSSh/6-311+G* calculations. The cluster symmetry and relative energy (in eV) are given at the below of each structure. The lowest energy structure is marked in blue. The inner atoms are marked in green.

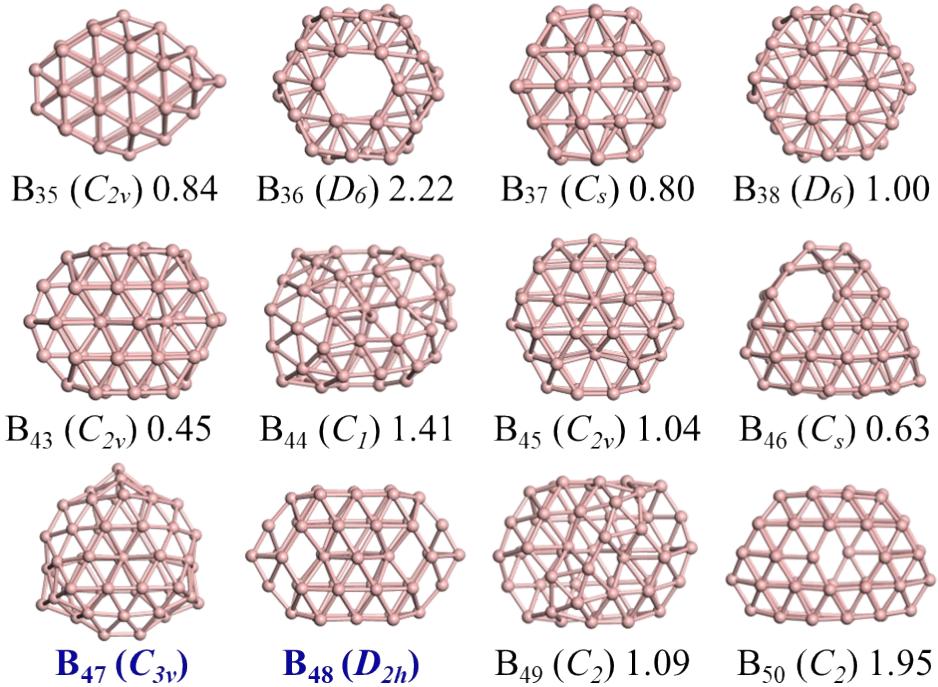


Fig. S4 The bilayer structures of neutral B_n ($n = 35\text{--}50$) clusters from TPSSh/6-311+G* calculations. The cluster symmetry and relative energy (in eV) are given at the below of each structure. The lowest energy structures are marked in blue.

Table S1. The energy differences of the representative isomers of B_n clusters ($n = 46, 48, 50$) in Fig. S1–S4 from TPSSh/6-311+G* and HSE06/6-311+G*.

Sizes	Isomers	ΔE (eV)	
		TPSSh/6-	HSE06/6-
B_{46}	I_Core-shell	0.00	0.00
	II_Quasi-planar	0.15	0.03
	IX_Bilayer	0.63	0.21
	XII_Cage	0.71	0.38
B_{48}	I_Bilayer	0.00	0.00
	IV_Core-shell	1.36	1.40
	VI_Quasi-planar	1.59	1.46
	VIII_Cage	1.69	1.15
B_{50}	I_Quasi-planar	0.00	0.00
	II_Core-shell	0.94	0.92
	IV_Cage	1.23	0.97
	XIV_Bilayer	1.95	1.94

Table S2(a) Structural motifs, point-group symmetries, relative energies ΔE , Binding energies (E_b) and HOMO-LUMO gap (E_{HL}) of the low-lying isomers of B_n ($n = 31\text{--}34$) clusters from TPSSh/6-311+G* calculations.

Sizes	Isomers	Symmetry	ΔE (eV)	E_b (eV)	E_{HL} (eV)
B_{31}	I Tubular	C_s	0.00	5.09	1.43
	II Quasi-Planar	C_s	0.05	5.08	1.51
	III Cage	C_2	0.29	5.08	1.75
	IV Quasi-Planar	C_I	0.30	5.08	0.97
	V Tubular	C_2	0.40	5.07	0.83
	VI Cage	C_I	0.41	5.07	1.55
	VII Cage	C_2	0.42	5.07	1.17
	VIII Tubular	C_s	0.47	5.07	1.11
	IX Cage	C_I	0.49	5.07	1.40
	X Cage	C_s	0.49	5.07	1.42
	XI Cage	C_I	0.54	5.07	1.07
B_{32}	I Tubular	D_{16d}	0.00	5.12	1.51
	II Quasi-Planar	C_s	0.64	5.11	1.65
	III Cage	C_2	0.91	5.10	2.23
	IV Tubular	C_s	0.98	5.10	1.43
	V Tubular	C_s	1.17	5.09	1.45
	VI Cage	C_2	1.23	5.09	1.55
	VII Cage	C_s	1.33	5.09	1.87
	VIII Cage	C_2	1.44	5.08	1.57
	IX Cage	C_s	1.46	5.08	1.81
	X Cage	C_s	1.71	5.08	1.65
B_{33}	I Quasi-Planar	C_s	0.00	5.11	0.79
	II Tubular	D_{11h}	0.05	5.10	0.82
	III Quasi-Planar	C_s	0.20	5.10	1.08
	IV Quasi-Planar	C_I	0.23	5.10	1.13
	V Tubular	C_s	0.26	5.10	1.61
	VI Quasi-Planar	C_I	0.42	5.09	1.05
	VII Cage	C_s	0.47	5.09	0.96
	VIII Cage	C_s	0.47	5.09	0.96
	IX Quasi-Planar	C_I	0.56	5.09	1.29
	X Tubular	C_s	0.56	5.09	1.20
B_{34}	XI Cage	C_2	0.57	5.09	2.08
	I Cage	C_2	0.00	5.13	1.97
	II Cage	C_s	0.09	5.12	2.02
	III Tubular	D_{17d}	0.16	5.12	0.45
	IV Cage	C_s	0.27	5.12	1.55
	V Quasi-Planar	C_I	0.29	5.11	0.78
	VI Cage	C_s	0.34	5.11	1.88
	VII Cage	C_I	0.53	5.11	1.79
	VIII Cage	C_I	0.63	5.11	1.80
	IX Quasi-Planar	C_{2v}	0.69	5.10	1.13
	X Tubular	C_s	0.77	5.10	1.10

Table S2(b) Structural motifs, point-group symmetries, relative energies ΔE , Binding energies (E_b) and HOMO-LUMO gap (E_{HL}) of the low-lying isomers of B_n ($n = 35\text{--}38$) clusters from TPSSh/6-311+G* calculations.

Sizes	Isomers	Symmetry	ΔE	E_b (eV)	E_{HL} (eV)
B_{35}	I Quasi-Planar	C_s	0.00	5.13	1.10
	II Cage	C_I	0.54	5.12	1.14
	III Cage	C_2	0.67	5.11	1.16
	IV Cage	C_I	0.76	5.11	1.07
	V Cage	C_2	0.81	5.11	1.04
	VI Cage	C_2	0.81	5.11	1.04
	VII Bilayer	C_{2v}	0.84	5.11	0.82
	VIII Bilayer	C_s	0.91	5.10	1.32
	IX Tubular	C_s	1.22	5.10	1.27
	X Cage	C_s	1.31	5.09	1.22
B_{36}	I Quasi-Planar	C_{6v}	0.00	5.17	1.50
	II Tubular	D_{12h}	0.87	5.14	0.51
	III Cage	C_s	1.41	5.13	1.48
	IV Bilayer	D_6	2.22	5.10	1.86
	V Cage	C_2	2.25	5.10	1.54
	VI Cage	C_2	2.31	5.10	1.32
	VII Tubular	C_s	2.42	5.10	1.08
	VIII Tubular	C_s	2.42	5.10	1.08
	IX Tubular	C_s	2.79	5.09	0.74
	X Bilayer	C_s	3.24	5.07	1.08
B_{37}	I Quasi-Planar	C_I	0.00	5.14	1.18
	II Quasi-Planar	C_I	0.22	5.13	1.23
	III Quasi-Planar	C_I	0.22	5.13	1.04
	IV Cage	C_s	0.35	5.13	1.33
	V Quasi-Planar	C_s	0.37	5.13	1.18
	VI Quasi-Planar	C_I	0.54	5.12	0.91
	VII Tubular	C_I	0.57	5.12	1.41
	VIII Tubular	C_I	0.57	5.12	1.44
	IX Quasi-Planar	C_I	0.58	5.12	0.95
	X Tubular	C_s	0.66	5.12	1.29
	XI Bilayer	C_s	0.80	5.12	1.12
B_{38}	I Quasi-Planar	C_s	0.00	5.15	0.72
	II Cage	C_I	0.48	5.14	1.83
	III Cage	D_{2h}	0.51	5.14	1.60
	IV Tubular	C_s	0.82	5.12	1.44
	V cage	C_s	0.89	5.12	0.95
	VI Bilayer	D_6	1.00	5.12	0.86
	VII cage	C_s	1.18	5.12	0.87
	VIII Tubular	C_s	1.31	5.11	1.01
	IX Tubular	C_s	1.33	5.11	0.98
	X Tubular	C_s	1.55	5.10	0.90
	XI Bilayer	D_{2h}	3.25	5.06	1.17

Table S2(c) Structural motifs, point-group symmetries, relative energies ΔE , Binding energies (E_b) and HOMO-LUMO gap (E_{HL}) of the low-lying isomers of B_n ($n = 39\text{--}42$) clusters from TPSSh/6-311+G* calculations.

Sizes	Isomers	Symmetry	ΔE	E_b (eV)	E_{HL} (eV)
B_{39}	I_Tubular	D_{13h}	0.00	5.17	0.78
	II_Core-Shell	O_h	0.18	5.17	0.64
	III_Quasi-Planar	C_s	0.29	5.16	1.19
	IV_Cage	C_3	0.98	5.14	2.23
	V_Cage	C_s	1.05	5.14	1.15
	VI_Cage	C_s	1.08	5.14	1.57
	VII_Cage	C_2	1.09	5.14	2.09
	VIII_Cage	C_2	1.09	5.14	2.09
	IX_Cage	C_2	1.09	5.14	2.09
	X_Core-Shell	C_{3v}	1.09	5.14	1.78
B_{40}	I_Cage	D_{2h}	0.00	5.19	2.40
	II_Cage	C_s	0.46	5.17	2.25
	III_Quasi-Planar	C_s	0.49	5.17	1.24
	IV_Quasi-Planar	C_s	0.70	5.16	1.31
	V_Cage	C_{2h}	0.93	5.16	1.96
	VI_Cage	C_l	1.22	5.15	2.13
	VII_Tubular	C_s	1.44	5.14	0.90
	VIII_Tubular	C_l	1.59	5.14	0.96
	IX_Tubular	C_s	1.65	5.14	1.16
	X_Core-Shell	C_{3v}	1.70	5.14	1.32
B_{41}	I_Quasi-Planar	C_s	0.00	5.17	0.55
	II_Cage	C_l	0.60	5.16	0.81
	III_Cage	C_s	0.60	5.16	0.98
	IV_Cage	C_s	0.66	5.16	1.56
	V_Cage	C_{2v}	0.94	5.15	0.90
	VI_Core-Shell	C_s	1.16	5.14	1.83
	VII_Core-Shell	C_s	1.27	5.14	1.11
	VIII_Cage	C_{2v}	1.38	5.14	0.81
	IX_Cage	C_{2v}	1.38	5.14	0.81
	X_Core-Shell	C_s	1.40	5.14	1.46
	XI_Core-Shell	C_s	1.41	5.14	1.22
	XII_Tubular	C_s	1.48	5.13	1.07
B_{42}	I_Tubular	D_{14h}	0.00	5.20	1.64
	II_Quasi-Planar	D_{7h}	0.59	5.18	1.39
	III_Cage	C_l	1.61	5.16	1.38
	IV_Cage	C_2	1.69	5.16	1.16
	V_Cage	C_{2v}	1.66	5.16	1.10
	VI_Cage	C_s	1.74	5.16	1.57
	VII_Cage	C_s	1.77	5.16	0.98
	VIII_Cage	C_3	1.87	5.16	2.02
	IX_Quasi-Planar	C_{2v}	1.95	5.15	0.91
	X_Cage	C_s	1.97	5.15	1.18
	XI_Core-Shell	C_l	2.15	5.15	1.23
	XII_Cage	C_2	2.46	5.14	0.73

Table S2(d) Structural motifs, point-group symmetries, relative energies ΔE , Binding energies (E_b) and HOMO-LUMO gap (E_{HL}) of the low-lying isomers of B_n ($n = 43\text{--}46$) clusters from TPSSh/6-311+G* calculations.

Sizes	Isomers	Symmetry	ΔE	E_b (eV)	E_{HL} (eV)
B_{43}	I Quasi-Planar	C_s	0.00	5.17	0.64
	II Tubular	C_s	0.18	5.16	0.90
	III Tubular	C_s	0.19	5.16	0.75
	IV Cage	C_s	0.20	5.16	0.92
	V Tubular	C_s	0.20	5.16	0.84
	VI Cage	C_{2v}	0.34	5.16	1.11
	VII Cage	C_s	0.36	5.16	1.07
	VIII Quasi-Planar	C_s	0.36	5.16	0.72
	IX Bilayer	C_{2v}	0.45	5.15	1.01
	X Quasi-Planar	C_l	0.57	5.15	0.93
	XI Core-Shell	C_s	0.66	5.15	1.09
B_{44}	I Quasi-Planar	C_s	0.00	5.17	1.15
	II Quasi-Planar	C_l	0.13	5.16	1.01
	III Quasi-Planar	C_s	0.21	5.16	0.74
	IV Cage	C_2	0.22	5.17	1.42
	V Cage	C_2	0.25	5.16	1.31
	VI Quasi-Planar	C_s	0.37	5.16	1.23
	VII Tubular	C_s	0.45	5.16	1.00
	VIII Cage	C_2	0.65	5.15	1.03
	IX Core-Shell	C_l	0.74	5.15	1.18
	X Cage	D_{2d}	0.77	5.15	1.15
	XI Tubular	C_2	1.02	5.14	0.93
	XII Bilayer	C_l	1.41	5.14	1.37
B_{45}	I Tubular	D_{15h}	0.00	5.19	0.77
	II Core-Shell	C_s	0.98	5.17	0.70
	III Bilayer	C_{2v}	1.04	5.17	0.75
	IV Quasi-Planar	C_l	1.19	5.17	0.87
	V Quasi-Planar	C_s	1.19	5.17	0.87
	VI Cage	C_2	1.27	5.17	1.37
	VII Core-Shell	C_s	1.37	5.16	1.07
	VIII Core-Shell	C_s	1.37	5.16	1.07
	IX Cage	C_{3v}	1.53	5.16	0.76
B_{46}	I Core-Shell	C_2	0.00	5.18	1.56
	II Quasi-Planar	C_{2v}	0.15	5.18	0.79
	III Tubular	C_l	0.45	5.17	0.79
	IV Quasi-Planar	C_s	0.48	5.17	0.57
	V Quasi-Planar	C_l	0.50	5.17	0.88
	VI Core-Shell	C_2	0.53	5.17	1.09
	VII Tubular	C_s	0.59	5.17	0.90
	VIII Core-Shell	C_2	0.61	5.17	1.28
	IX Bilayer	D_{2h}	0.63	5.17	1.27
	X Bilayer	C_s	0.65	5.17	1.37
	XI Bilayer	C_s	0.65	5.17	1.37
	XII Cage	C_2	0.71	5.17	1.48

Table S2(e) Structural motifs, point-group symmetries, relative energies ΔE , Binding energies (E_b) and HOMO-LUMO gap (E_{HL}) of the low-lying isomers of B_n ($n = 47\text{--}50$) clusters from TPSSh/6-311+G* calculations.

Sizes	Isomers	Symmetry	ΔE	E_b (eV)	E_{HL} (eV)
B_{47}	I Bilayer	C_{3v}	0.00	5.18	1.50
	II Quasi-Planar	C_s	0.17	5.17	1.09
	III Core-Shell	C_3	0.30	5.17	1.27
	IV Cage	C_s	0.49	5.17	1.04
	V Cage	C_{2v}	0.51	5.17	0.79
	VI Cage	C_{2v}	0.66	5.17	1.27
	VII Tubular	C_s	0.69	5.16	0.58
	VIII Cage	C_s	0.73	5.16	0.88
	IX Cage	C_s	0.73	5.16	0.88
B_{48}	I Bilayer	D_{2h}	0.00	5.21	0.92
	II Tubular	D_{16h}	0.50	5.19	0.46
	III Bilayer	C_{2v}	1.09	5.18	1.39
	IV Core-Shell	C_2	1.36	5.18	1.09
	V Bilayer	C_{2v}	1.45	5.17	1.34
	VI Quasi-Planar	C_l	1.59	5.17	1.15
	VII Quasi-Planar	C_s	1.63	5.17	0.71
	VIII Cage	C_2	1.69	5.17	1.52
	IX Quasi-Planar	C_l	1.73	5.17	0.91
	X Cage	C_2	1.76	5.17	1.32
	XI Cage	C_s	1.76	5.17	0.96
B_{49}	I Quasi-Planar	C_{2v}	0.00	5.20	0.91
	II Cage	C_{2v}	0.96	5.19	0.57
	III Core-Shell	C_{2v}	1.09	5.18	1.62
	IV Bilayer	C_2	1.09	5.18	1.08
	V Bilayer	C_s	1.21	5.18	1.03
	VI Cage	C_{2v}	1.32	5.18	1.39
	VII Tubular	C_s	1.36	5.17	0.78
	VIII Tubular	C_l	1.41	5.17	0.56
B_{50}	I Quasi-Planar	C_{2v}	0.00	5.21	1.28
	II Core-Shell	C_2	0.94	5.19	0.88
	III Core-Shell	C_2	1.13	5.18	1.04
	IV Cage	C_{2v}	1.23	5.18	0.88
	V Quasi-Planar	C_2	1.34	5.18	0.90
	VI Cage	D_{6d}	1.47	5.18	1.55
	VII Cage	D_2	1.59	5.17	1.05
	VIII Cage	C_2	1.65	5.17	1.17
	IX Tubular	C_s	1.67	5.17	0.56
	X Quasi-Planar	C_s	1.70	5.17	1.11
	XI Cage	C_2	1.74	5.17	1.05
	XII Cage	C_2	1.76	5.17	0.90
	XIII Cage	C_{2h}	1.77	5.17	0.86
	XIV Bilayer	C_2	1.95	5.16	0.90

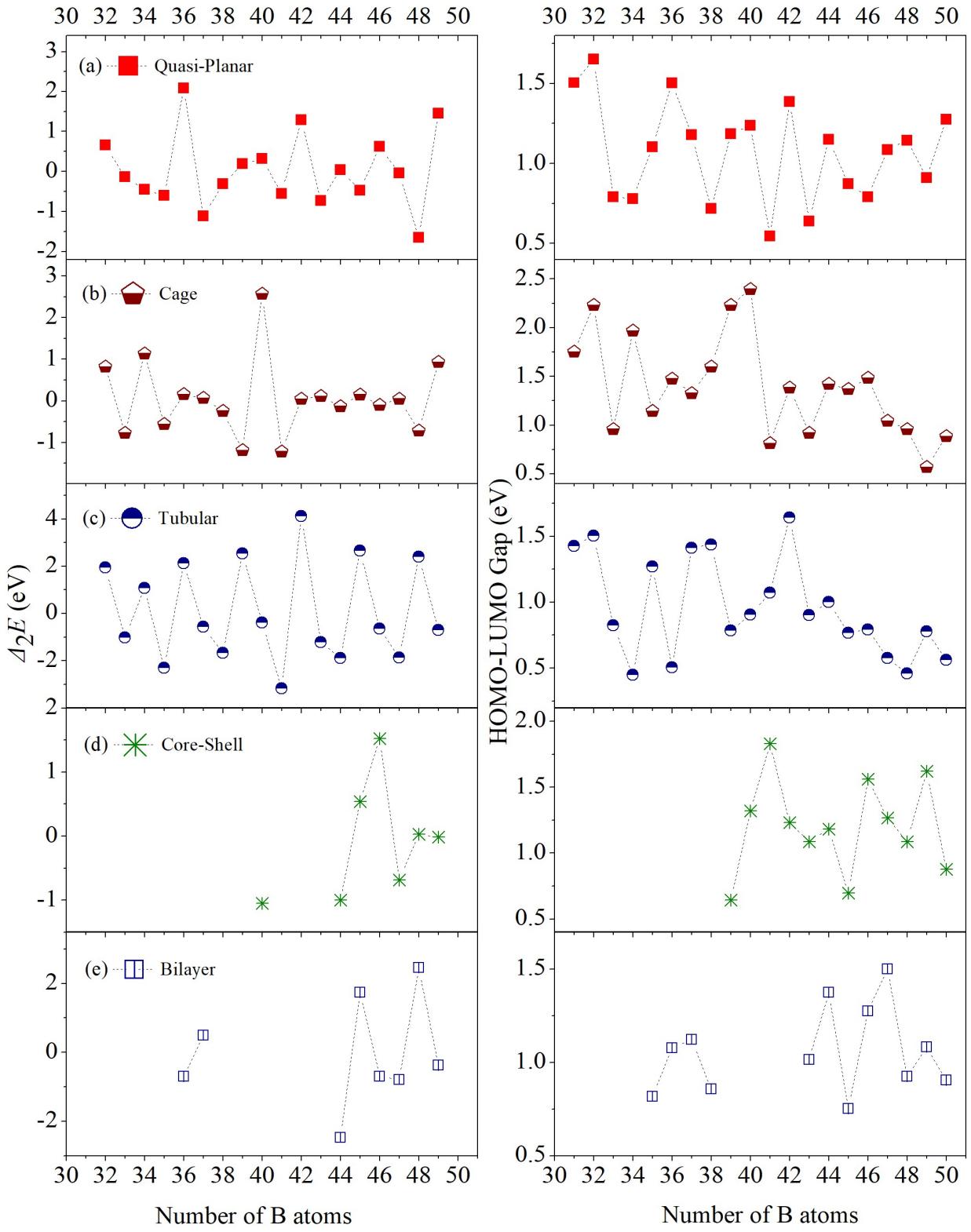


Fig. S5 Second order energy difference of different structural patterns ($\Delta_2 E$) and HOMO-LUMO gap (E_{HL}) of B_n ($n = 31$ – 50) cluster: (a) quasi-planar, (b) cage, (c) tubular, (d) core-shell and (e) bilayer structures.

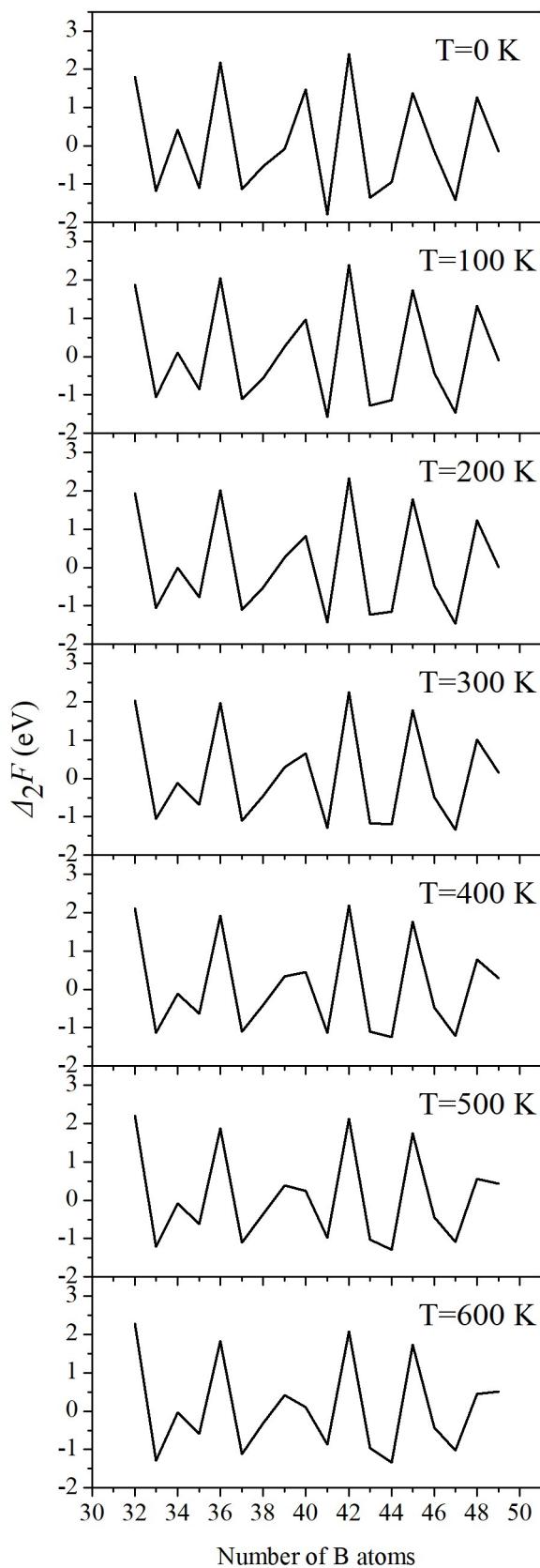


Fig. S6 Second order difference of Helmholtz free-energy ($\Delta_2 F$) of B_n ($n = 31\text{--}50$) cluster at different temperatures.

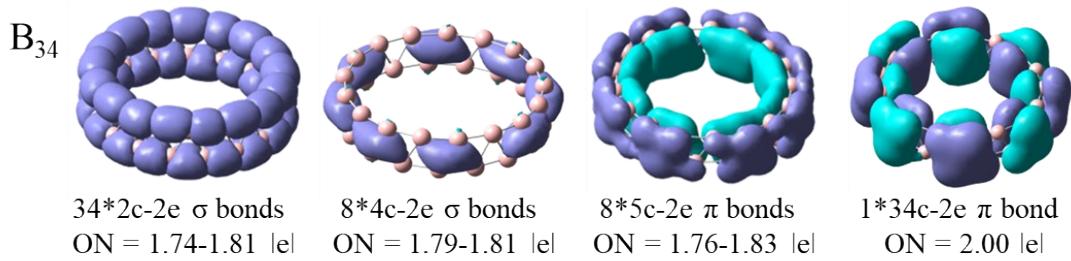


Fig. S7. AdNDP chemical bonding analyses of the double-ring tubular B₃₄. ON denotes occupation number.

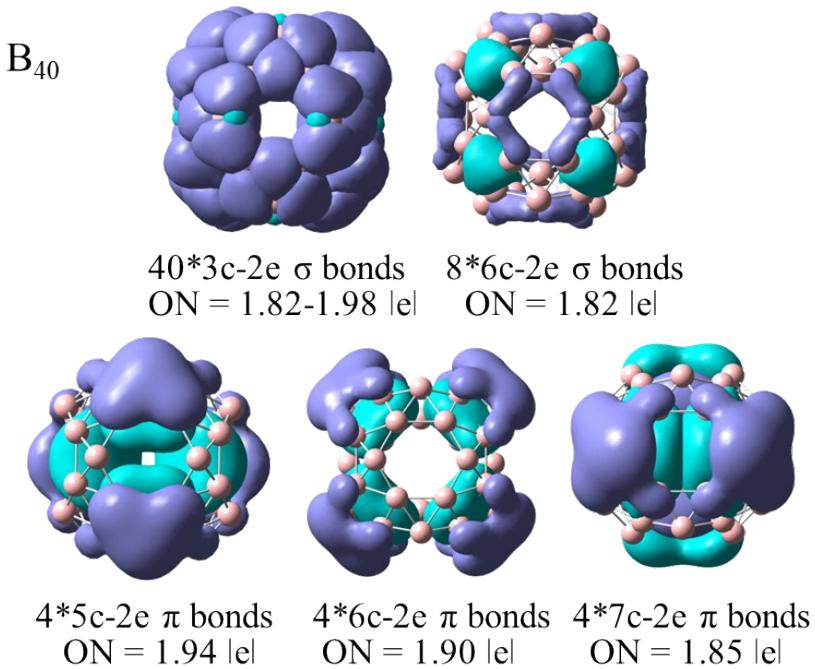


Fig. S8. AdNDP chemical bonding analyses of the B₄₀ cage. ON denotes occupation number. The AdNDP analysis identifies 40 3c-2e σ bonds on the B₃ triangles and eight 6c-2e σ bonds, three different π systems: four 5c-2e bonds, four 6c-2e bonds and four 7c-2e bonds distributed evenly over the cage surface along the interwoven BDCs.

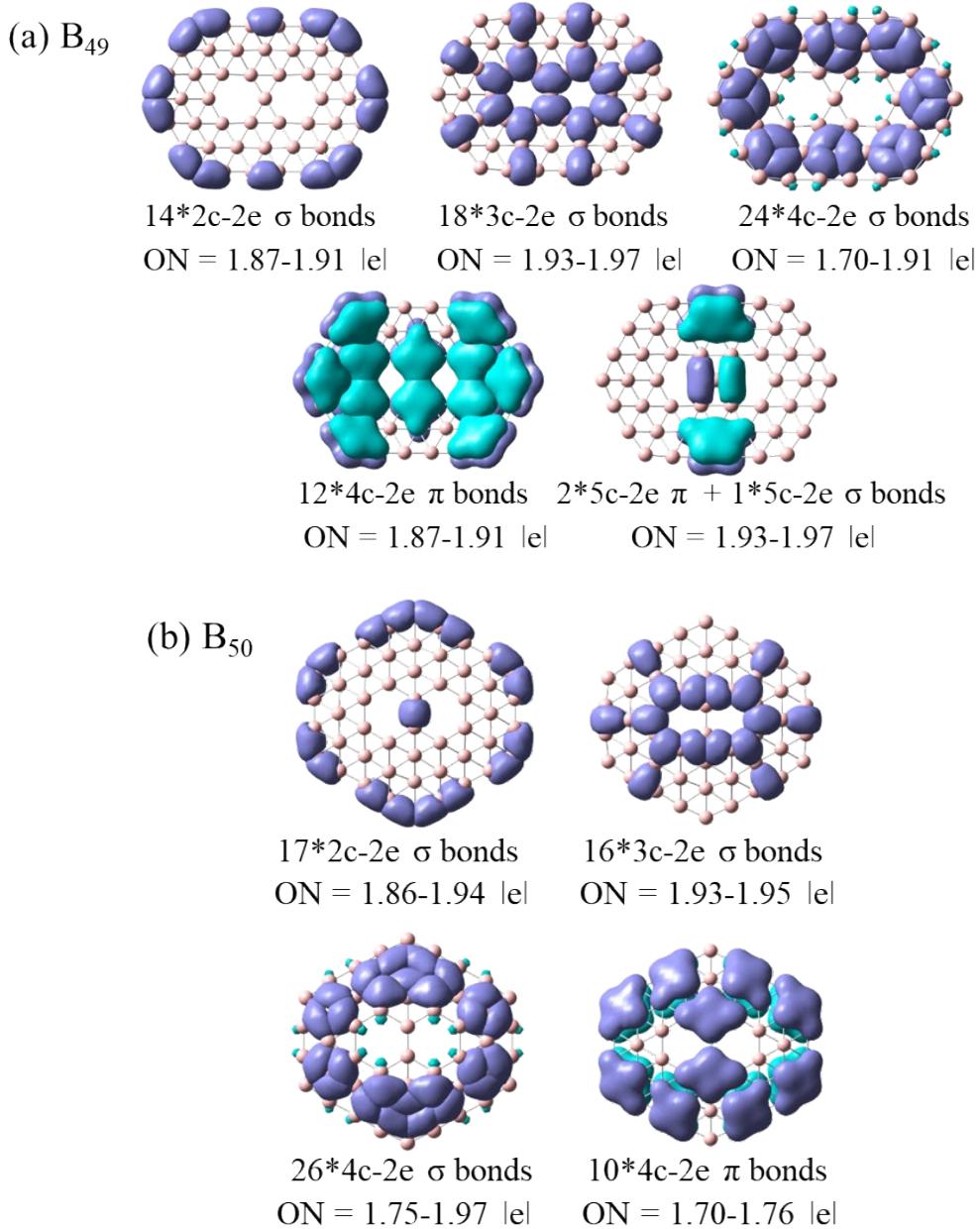


Fig. S9. AdNDP chemical bonding analyses of the quasi-planar for their closed-shell (a) B_{49}^- and (b) B_{50} clusters. ON denotes occupation number.