Figure S1. The shear stress-strain curves obtained from 2D diluted triangular networks made up of linear elastic fibers. The coordination number $z$ is about 3.1 in all networks but different values of $\bar{\kappa}$ are considered. The critical strain $\gamma_0$, shown by red circles, denotes the onset of nonlinearity in the overall mechanical response. The top right inset shows the differential shear stiffness $K$ versus the applied shear strain and the top left inset represents the variation of $K' = d(\log(K))/d(\log(\gamma))$; the maximum point of this plot, denoted by the star symbol, is the inflection point of the $\log(K)$ versus $\log(\gamma)$ curve and gives an estimate for critical strain $\gamma_c$.

The total bending energy $H_b$ and total stretching energy $H_s$ in random fiber networks are, respectively, given as $H_b = \sum_{\text{fibers}} \frac{K}{2} \int \left| \frac{df}{ds} \right|^2 ds$ and $H_b = \sum_{\text{fibers}} \frac{\mu}{2} \int \left( \frac{dl}{ds} \right)^2 ds$ where $\frac{dl}{ds}$ is the change in length and $\left| \frac{df}{ds} \right|$ is the curvature of a fiber. The relative contributions of bending and stretching energy in networks composed of linear elastic fibers are shown in Figure S2 where $H = H_s + H_b$. 
Figure S2. The relative contributions of bending and stretching energy in networks composed of linear elastic fibers is plotted as a function of the applied shear strain. The bending energy is dominant for strains less than critical shear strain $\gamma_c$. When $\gamma > \gamma_c$, the stretching energy becomes important.
Figure S3. The variation of the differential shear modulus of random fiber networks versus the shear stress for linear elastic fibers with different bending rigidity $\bar{k} = 0.00001 - 0.1$. The shear modulus and stress are plotted in units of $\mu/l$. Three different regions are observed. Initially, the network stiffness is independent of the stress. With increasing the stress, the stiffness increases as $K \propto \sigma^\alpha$ where $\alpha$ varies from 0.6 to 1.5 as $\bar{k}$ decreases from $10^{-1}$ to $10^{-5}$ (shown in the inset). With further increase of the stress, the stiffness becomes independent of stress (see the red dashed lines); this is a limitation of linear elastic models and does not agree with experimental measurements where the network stiffness increases until failure.