Figure S1 Diffuse reflectivity spectra of (A) of europium bifunctional hybrids and (B) of terbium bifunctional hybrids
The calculation of different parameters for the luminescence quantum efficiency

$A_r$ can be obtained by summing over the radiative rates $A_{0j}$ for each $^5D_0 \rightarrow ^7F_J$ ($J = 0–2$) transitions of Eu$^{3+}$:

$$A_{\text{rad}} = \sum A_{0j} = A_{00} + A_{01} + A_{02} \quad (1)$$

Since the branching ratio for the $^5D_0 \rightarrow ^7F_{3,4,5,6}$ transitions are too weak to be detected experimentally, so they can be neglected and their influence can also be ignored in the depopulation of the $^5D_0$ excited state. The magnetic dipole $^5D_0 \rightarrow ^7F_1$ is considered as an internal reference for its independence of the chemical environments around Eu$^{3+}$ and $A_{00}$, the experimental coefficients of spontaneous emission, are calculated according to the equation.

$$A_{0j} = A_{01}(I_{0j}/I_{01})(\nu_{01}/\nu_{0j}) \quad (2)$$

Here $\nu_{0j}$ refers to the energy barycenter ($J = 0, 1, 2$), which can be determined as the reciprocal of the wavelength where the emission peaks of Eu$^{3+}$'s $^5D_0 \rightarrow ^7F_J$ emission transitions. $A_{01}$, the Einstein’s coefficient of spontaneous emission ($^5D_0 \rightarrow ^7F_1$), can be determined to be 50 s$^{-1}$ approximately ($A_{01} = n^3A_{01}$ (vacuum)) in vacuum.

Since the lifetime and the $I_{02}/I_{01}$ (red/orange ratio) are the two mainly factors to determine the value $\eta$, so the longer lifetimes and bigger red/orange ratio, the higher the quantum efficiency. The relation among the radiative ($A_r$), nonradiative ($A_{nr}$) transition rates and lifetime ($\tau$) can be described as the following equation

$$\tau_{\exp} = (A_r + A_{nr})^{-1} \quad (3)$$

According to the radiative transition rate constant and experimental luminescence lifetime, the quantum efficiency can be calculated from the following equation:

$$\tau = A_r\tau_{\exp} \quad (4)$$