S1. Surface profiles of Sb$_2$Te$_3$ and Bi$_2$Te$_3$ films

Figure S1: (a) surface profile of a 6 µm Sb$_2$Te$_3$ film on Al$_2$O$_3$ substrate, (b) surface profile of 6 µm Bi$_2$Te$_3$ film on Al$_2$O$_3$ substrate.

S2. Measurement set up

Figure S2: Image of the system set-up used for measuring the voltage from the thermopower experiments. Ignition takes place at one end of the fuel/thermoelectric film/Al$_2$O$_3$ sample and the reaction wave self-propagates to the other end resulting in a voltage signal.

S3. Oscillation dependance on $\beta$ value

Figure S3: Region in the parameter ($\beta$, $\ell$/$\beta$) space where combustion waves and extinction occurs. Figure reproduced from reference (1S).

S4. Thermal conductivity comparison of the substrates

In order to show the difference in thermal conductivities of the Al$_2$O$_3$ and terracotta substrates, both were heated at one end for the same duration using a custom-made blowtorch and the temperature profile was obtained at the other end to see how fast thermal conduction occurs through the substrate. In Fig. S1, we can observe that temperature for Al$_2$O$_3$ rises rapidly to about 130 °C in about 40 sec. The terracotta substrate however reaches a maximum temperature of around 40 °C and takes about twice the time taken by Al$_2$O$_3$ to reach peak temperature. We note that the temperature continues to rise even after the heat source is withdrawn. This is because heat takes time to travel from one end to the other and finally dissipates, depending on the thermal conductivity of the material. Therefore, the peak temperature is obtained a few seconds after the heat source is turned off. However, we can see that terracotta takes approximately 60 seconds longer than Al$_2$O$_3$ to reach its peak temperature. A similar observation is made in the cooling durations of the two substrates. While Al$_2$O$_3$ cools down to room temperature in about 150 seconds from the point of its peak temperature, terracotta takes approximately 240 seconds to cool down to room temperature. These observations clearly demonstrate that the thermal conduction through terracotta is significantly slower than Al$_2$O$_3$. Also it further supports our experimental results, which show that the reaction velocities for Al$_2$O$_3$ substrates are considerably higher than terracotta. This endorses our approximation that the thermal conduction takes place through the substrate.

S4.1 Calculation of specific power

In order to obtain the specific power, the masses of the fuel (nitrocellulose & sodium azide) and thermoelectric films were considered. The relationship between mass, density and volume was used for calculating the mass of the Sb$_2$Te$_3$ and Bi$_2$Te$_3$ films as: $m = \rho \times V$, where, $m$ is the mass in grams (g), $\rho$ is the density in g.cm$^{-3}$ and $V$ is the volume in cm$^3$. $\rho$=6.5 g.cm$^{-3}$ and 7.7 g.cm$^{-3}$ for Sb$_2$Te$_3$ and Bi$_2$Te$_3$ respectively. ($3S,4S$) and $V$ depends on the dimensions of the sample and is calculated using the formula, $V = l \times w \times h$. 
where \( l \) is the length, \( w \) is the width and \( h \) is the thickness of the thermoelectric film in cm, respectively.

For fuels, nitrocellulose and NaN\(_3\) in our case, we knew the concentration of the two in their respective solutions. As mentioned in Section 2.2, 15g.L\(^{-1}\) nitrocellulose and 20g.L\(^{-1}\) NaN\(_3\) were used by drop casting. Hence, we know the amount of solution for each fuel and therefore the corresponding mass for each sample, depending on the volume of the liquid placed on the thermoelectric films. The sum of the masses of the thermoelectric film and the fuels gives the total mass of the materials used. The corresponding power from the sample is then scaled to per kg of the materials to obtain the specific power of the sample in kW kg\(^{-1}\).

**Note:**

The mass of Al\(_2\)O\(_3\) has not been considered as its effect was negligible compared to the masses of Sb\(_2\)Te\(_3\)/Bi\(_2\)Te\(_3\) and the fuels. The entire thickness of 100 µm does not contribute to the thermal conduction. As can be seen from Fig. S4, it takes 25 sec for the heat to be thermally conducted through 100 µm of Al\(_2\)O\(_3\) from one side to another. Taking the short thermopower reaction time [Fig. 3(b)] of approximately 0.5 sec as an example (most of them are shorter than this), we can therefore estimate that only a 2 µm thick Al\(_2\)O\(_3\) substrate will suffice for the purpose of the thermal conduction in these experiments. Even assuming an extreme effective thickness of 5 times larger than 2 µm (10 µm) for Al\(_2\)O\(_3\) from one side to another, the heat source was kept in place until the temperature at the other end reached a constant value (approximately 200 s) in equilibrium with the ambient. Figure S5(b) shows the temperature profile for the two samples. It shows that the temperature rises to approximately 47ºC for both samples. However, the rate of rise is approximately 2.2 times higher for the Al\(_2\)O\(_3\)/Sb\(_2\)Te\(_3\) sample which is analogous to the ratio of the thermal conductivities of Sb\(_2\)Te\(_3\) and Bi\(_2\)Te\(_3\). This result endorses our explanation that the rate of heat conduction in Sb\(_2\)Te\(_3\) is ~2.5 times faster than that in Bi\(_2\)Te\(_3\); based on the ratio of their thermal conductivity.

**Figure S5:** a) Measurement set up, b) Difference in thermal conduction of Sb\(_2\)Te\(_3\) and Bi\(_2\)Te\(_3\) films

The thermal diffusivity (\( \alpha \)) is defined as follows:

\[
\alpha = \frac{k}{\rho c_p} \quad \text{(S1)}
\]

where, \( k \) is the thermal conductivity, \( \rho \) is the density and \( c_p \) is the specific heat. The density of Sb\(_2\)Te\(_3\) and Bi\(_2\)Te\(_3\) is approximately 6.5 g/cm\(^3\) and 7.7 g/cm\(^3\), respectively.\(^{(6S,7S)}\)

The specific heats of Sb\(_2\)Te\(_3\) and Bi\(_2\)Te\(_3\) are 0.19 J/g.K and 0.16 J/g.K respectively.\(^{(6S,7S)}\) Therefore, the product \( \rho c_p \) in the denominator is 1.235 and 1.232 for Sb\(_2\)Te\(_3\) and Bi\(_2\)Te\(_3\) respectively, which are almost equal. As a result it can be said that the thermal diffusivity (\( \alpha \)) is directly proportional to the thermal conductivity \( \kappa \) in the case of our experiments.

Figure S5 (a) shows the measurement set up. The Sb\(_2\)Te\(_3\)/Al\(_2\)O\(_3\) and Bi\(_2\)Te\(_3\)/Al\(_2\)O\(_3\) samples of ~12 mm length were used for the measurement and the thickness of both Sb\(_2\)Te\(_3\) and Bi\(_2\)Te\(_3\) films were equal to 6 µm. A heater with a sharp tip was used as the heating source and was positioned at one end on the sample. The temperature profile was acquired at the opposite end using a TENMA 727730 thermocouple. The heat source was kept in place until the temperature at the other end reached a constant value (approximately 200 s) in equilibrium with the ambient. Figure S5(b) shows the temperature profile for the two samples. It shows that the temperature rises to approximately 47ºC for both samples. However, the rate of rise is approximately 2.2 times higher for the Al\(_2\)O\(_3\)/Sb\(_2\)Te\(_3\) sample which is analogous to the ratio of the thermal conductivities of Sb\(_2\)Te\(_3\) and Bi\(_2\)Te\(_3\). This result endorses our explanation that the rate of heat conduction in Sb\(_2\)Te\(_3\) is ~2.5 times faster than that in Bi\(_2\)Te\(_3\); based on the ratio of their thermal conductivity.

Heat conduction is defined as

\[
H = \frac{\Delta Q}{\Delta t} = kA\frac{\Delta T}{x}
\]

where \( \frac{\Delta Q}{\Delta t} \) is the rate of heat flow, \( k \) is the thermal conductivity, \( A \) is the total cross sectional area of the conducting surface, \( \Delta T \) is the temperature difference and \( x \) is the thickness of the conducting surface separating the two temperatures. As the dimensions, thickness, and temperature difference of our Sb\(_2\)Te\(_3\) and Bi\(_2\)Te\(_3\) samples are similar the thermal conductivity is the most dominant factor affecting the rate of heat conduction. Hence, we believe that our assumption in the paper is consistent with the theory. Also experimental results of Figure S5 shown in the supplementary prove the same.\(^{(7S)}\)

**S6. Comparison of electrical conductivities of Sb\(_2\)Te\(_3\) and Bi\(_2\)Te\(_3\)**

In order to compare the electrical conductivities of the two films four point probe measurements were carried out on Sb\(_2\)Te\(_3\) and Bi\(_2\)Te\(_3\). The following equation was then used to calculate the resistivity:
\[ \rho = \frac{R_{\text{Sh}}}{t}; \]  
(S2)

in which \( \rho \) is the resistivity of the film in \( \Omega \cdot \text{m} \), \( R_{\text{Sh}} \) is the sheet resistance of the film in \( \Omega/\) and \( t \) is the thickness of the film in \( \text{m} \).

**Sb\textsubscript{2}Te\textsubscript{3} film**

The four point probe showed that the sheet resistance of the Sb\textsubscript{2}Te\textsubscript{3} film was 3.103 \( \Omega/\) for the Sb\textsubscript{2}Te\textsubscript{3} film of approximately 6\( \mu \text{m} \). Therefore the resistivity of the film was calculated to be \( 1.86 \times 10^{-5} \Omega \cdot \text{m} \). Inverse of the resistivity gives the conductivity.

Thus, the electrical conductivity of the Sb\textsubscript{2}Te\textsubscript{3} film was found to be \( 5.3 \times 10^{4} \text{S/m} \).

**Bi\textsubscript{2}Te\textsubscript{3} film**

A similar approach was used to calculate the electrical conductivity of the Bi\textsubscript{2}Te\textsubscript{3} film. The sheet resistance of the Bi\textsubscript{2}Te\textsubscript{3} film was found to be 1.75 \( \Omega/\). Therefore the electrical conductivity of the film was calculated to be \( 1.1 \times 10^{5} \text{S/m} \).

The above measurements show that the electrical conductivity of Bi\textsubscript{2}Te\textsubscript{3} is an order of magnitude higher than Sb\textsubscript{2}Te\textsubscript{3}. The values obtained are also comparable with the literature values of \( 10^{4} \) and \( 10^{5} \text{S/m} \) for Sb\textsubscript{2}Te\textsubscript{3} and Bi\textsubscript{2}Te\textsubscript{3} films respectively. (8S-10S)

**References for Supplementary**


