Supplementary movie 1: Image sequences of droplets flowing into a test section containing an array of holes. Droplets become anchored in the array of holes. The oil flow rate is 2 μL/min. The movie corresponds to 200 seconds of experiment time (1 frame per second). The scale bar represents 200 μm.

Supplementary movie 2: A train of droplets of radius R = 180 μm flowing from left to right in the test section of height h = 50 μm over a hole of diameter d = 75 μm. In this configuration, the critical oil flow rate below which a single droplet remains anchored is Q_c = 16.5 μL/min. **Top:** The oil flow rate is set to Q_o = 12 μL/min. The train of droplets enters a buffering mode in which droplets are temporarily anchored to the hole until the following ones in the train bump them out of the hole. **Bottom**: The oil flow rate is lowered to Q_o = 6μL/min. The train of droplets enters a parking mode in which a single droplet is permanently anchored to the hole and resists collisions with other droplets. The movie corresponds to 75 seconds of experiment time (2 frames per second).

Supplementary movie 3: Water drops guided from left to right along a microfabricated sinusoidal rail (50 μm in width and 50 μm in depth) in an external flow of Q_o = 18μL/min. The movie corresponds to 100 seconds of experiment time (2 frames per second).

Supplementary movie 4: Polarization microscopy image sequences of anchored droplets containing sickle red blood cells in a stream of oxygenated oil (top) and deoxygenated oil (bottom) recorded simultaneously in the droplet array device shown in the inset. Images were taken with a 10 X objective and the background was subtracted. Using the notation defined in Figure 5a: $Q_1 = Q_2$ = 2 μL/min and the partial pressure of oxygen is equal to $C_1 \approx$ 21 kPa and $C_2 \approx$ 0 kPa. The inset shows the relative droplet position in the channel. The movie corresponds to 25 seconds of experiment time (2 frames per second). The scale bar represents 100 μm.

Supplementary movie 5: Polarization microscopy image sequence showing the polymerization of intracellular hemoglobin of red blood cells in a linear array of anchored droplets as deoxygenated oil flows from left to right replacing oxygenated oil in the test region. Images were taken with a 4 X objective and the background was subtracted. The contour of the hole is outlined in blue. The oil flow rate is 1 μL/min. The scale bar represents 200 μm.

Supplementary movie 6: Polarization microscopy image sequence showing the polymerization and depolymerization of intracellular hemoglobin in sickle red blood cells due to the alternating flows of the two oil channels. Images were taken with a 4 X objective and the background was subtracted. The contour of the hole is outlined in blue. The partial pressures of oxygen of the oil channels are C_1 = 21 kPa and C_2 = 0 kPa. The flow changes are the following t = 5 s: Q_1 = 1 → 0 μL/min, Q_2 = 0 → 1 μL/min, t = 155 s: Q_1 = 0 → 1 μL/min, Q_2 = 1 → 0 μL/min, t = 225: Q_1 = 1 → 0 μL/min, Q_2 = 0 → 1 μL/min. The scale bar represents 100 μm.