Appendix

In this supplementary document to the article, detailed information about our the derivation of our equations of motion, methods for obtaining analytical results in the adiabatic limit, and a brief description of the movies are provided.

A1. Theory

As shown in Figure 1 in the article, we consider a suspension of superparamagnetic beads exposed to a square array of ferromagnetic disks (with lattice period $d$) that are identical both in size and magnetization. In order to simplify the magnetic field calculation, we treat each disk as a pair of opposite magnetic point poles separated by the disk diameter, i.e., $d_M$. The magnetic pole distribution of the micro-magnet array can be expressed by Eq. I, where $\lambda_0$ is the effective magnetic pole density. The Fourier expansion of Eq. I will yield Eq. 1 in the article.

$$
\lambda(x, y) = -\lambda_0 \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \left[ \delta(x - nd) - \delta(x - (nd + d_M)) \right] \delta(y - md) \quad (1)
$$

The magnetic scalar potential is governed by Laplacian equation and can be solved by separation of variables. Taking the negative gradient of the scalar potential and including the oscillating field, $\vec{H}(t) = H_0 \left[ \hat{x} \sin(\omega t) + \hat{z} \sin(\omega t + \phi_0) \right]$, the expression of the total magnetic field as Eq. 2 is obtained.

$$
\vec{B}_M = \mu_0 \vec{m} \cdot \nabla \vec{H}_{tot},
$$

with the forcing magnitude $F_0 = 2\pi \mu_0 \vec{m} \cdot \nabla H_0 \lambda_0 / d$ and using the magnetic susceptibility of the bead as $\chi = 3 - \chi / (3 + \chi)$ which is consistent with a spherical, linearly magnetizable bead. The x component of the above equation yields Eq. 3.

$$
F = F_0 \left[ \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \frac{n}{N} \sin(\omega t) u_n(\xi_x) - \sin(\omega t + \phi_0) v_n(\xi_x) \right] n \cdot \cos(m \xi_x) e^{-N \xi_x}
$$

$$
\delta \left[ \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \frac{n}{N} \sin(\omega t) v_n(\xi_x) + \sin(\omega t + \phi_0) u_n(\xi_x) \right] m \cdot \sin(m \xi_x) e^{-N \xi_x}
$$

$$
\delta \left[ \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \frac{n}{N} \sin(\omega t) u_n(\xi_x) + \sin(\omega t + \phi_0) v_n(\xi_x) \right] N \cdot \cos(m \xi_x) e^{-N \xi_x}
$$

A2. Asymptotic Analysis

The equation of motion for the bead is obtained from the instantaneous balance between magnetic force and fluid drag. We write this in dimensionless form as:

$$
\ddot{\xi}_x = \omega_0 \sum_{n=1}^{\infty} \sum_{m=-\infty}^{\infty} \left[ \frac{n}{N} u_n(\xi_x) \sin(\omega t) - v_n(\xi_x) \sin(\omega t + \phi_0) \right]
$$

where $\omega_0 = 2\pi \sqrt{d / \eta a^3}$ and $a = 16 \pi^2 \mu_0 \chi / 9 \eta d^2$. As shown in the article, the adiabatic solution can be derived by considering the limit of extremely low driving frequencies, in which we can assume the...
bead’s velocity approaches $\dot{\xi} = 0$. This approach allows us to derive a direct analytical relationship for the bead as a function of time, which is given as:

$$\sin(\omega t) = \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} n \cdot u_n(\xi_n)$$

When we consider the simplest possible periodically charged substrate with monochromatic, unidirectional periodicity ($n=1$ and $m=0$), and using phase $\phi_0=\pi/2$ and $dM=d/2$, we arrive at expression (7) in the article.

A3. Brief Description of Supplementary Movies

Eleven video clips are submitted with the article to demonstrate the numerical and experimental results.

SI-1 provides an animation of the time modulated potential energy landscape superimposed on the calculated position of the superparamagnetic for the simulated conditions of $\omega x = \pi$ rad/s and $R_f=7/5$, corresponding to Figure 1b in the manuscript.

SI-2 shows the open trajectories of the bead when $R_f$ is 1/1. (Figure 3[A] in the manuscript)

SI-3 shows the open trajectories of the bead when $R_f$ is 3/1. (Figure 3[B] in the manuscript)

SI-4 shows the open trajectories of the bead when $R_f$ is 7/5. (Figure 3[E] in the manuscript)

SI-5 shows the open trajectories of the bead when $R_f$ is 9/5. (Figure 3[F] in the manuscript)

SI-6 shows the open trajectories of the bead when $R_f$ is 5/3. (Figure 3[C] in the manuscript)

SI-7 shows the open trajectories of the bead when $R_f$ is 7/3. (Figure 3[D] in the manuscript)

SI-8 shows the closed trajectories of the bead when $R_f$ is 49/50. (Figure 3[J] in the manuscript)

SI-9 shows the closed trajectories of the bead when $R_f$ is 2/1. (Figure 3[I] in the manuscript)

SI-10 demonstrates multiplexed motion in which the small beads move but the big beads do not, which occurs at a phase of $\phi_v=150^\circ$.

SI-11 demonstrates multiplexed motion in which the big beads move but the small beads do not, which occurs at a phase of $\phi_v=158^\circ$.

References