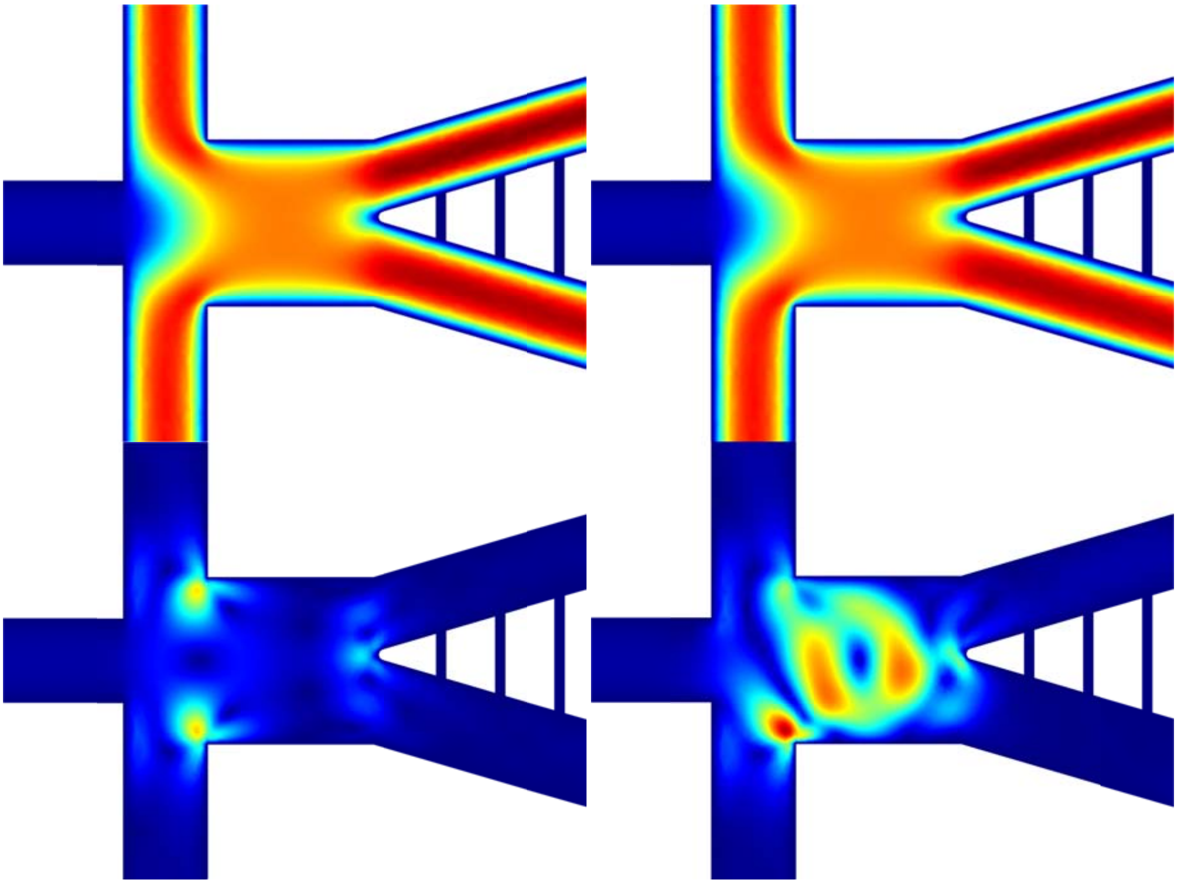

Supplementary Figure 1: Inertial effects

(top left) Inertial Navier-Stokes simulation of pressure-driven Poiseuille flow

(top right) Non-inertial Stokes simulation of pressure-driven Poiseuille flow

(bottom left) Difference between inertial Navier-Stokes flow and non-inertial Stokes flow for pressure driven Poiseuille flow: Inertial effects are minor and only occur at sharp corners.

(bottom right) Difference between inertial Navier-Stokes flow and non-inertial Stokes flow for SAW-actuated pressure driven Poiseuille flow: large inertial effects occur at the center of the channel that are caused by acoustically-actuated flow

Supplementary Discussion 1: Inertial Effects

In microfluidics, the inertial term of the Navier-Stokes Equation is usually neglected. The resulting Stokes Equation is fully linear.

To elucidate this question, we carried out comparative simulations of our channel: once with inertial Navier-Stokes flow and once with simple Stokes flow.

First, the pressure driven flow without SAW was simulated. Suppl. fig. 1 shows the velocities in the central xy-plane of the two simulations side by side (top), and the velocity difference of the two simulations (bottom left). For most of the channel, there are no deviations due to inertial flow. Only in two regions near the edges of the sheath flow inlet there is a small difference of 11mm/s which is about 10% of actual fluid velocity.

The picture changes when acoustics are added, see suppl. fig. 1 (bottom right). In the region of the SAW vortex inertial effects obviously play an important role, with a maximum difference of 16mm/s.

The reason for this SAW-triggered inertial effects can be understood from the inertial term $(\boldsymbol{v}\nabla)\boldsymbol{v}$ of the Navier-Stokes Equation (11). The term describes advective acceleration, i.e. the directional derivative of the flow velocity in flow direction. In one-dimensional linear flow, e. g. Hagen-Poiseuille Flow through a pipe, this term is zero because velocities along flow lines are constant. This holds also for linear microchannels. However, at turns (e. g. near the edge of the sheath flow inlet), the inertial term is non-zero and generates a zentrifugal force. The introduction of a SAW-generated volume force perpendicular to the flow direction can create a non-zero inertial term even in linear channels. Therefore, for SAW-driven flows, the full inertial Navier-Stokes equation has to be solved.

The inertial term is also important for the generation of SAW streaming. When deriving the equations of motion for an acoustic wave, 2nd order terms in velocity u are usually neglected. This leads to the non-inertial compressible Stokes Equation, which is solved by a dampened longitudinal wave with velocity $u_z(z,t) = u_0 \exp(-bz) * \cos(\omega t - kx)$. However, if 2nd order effects are not neglected, the full Navier-Stokes Equation has to be solved. The inertial term $u_z \partial_z u_z$ has to be balanced by an additional time-independent volume force $f = -\, b\rho u_0^2 \exp(-2bz)$, which is the source of acoustic streaming. Due to the different timescales of high-frequency acoustic wave and stationary flow, the two can be calculated separately. Ultimately, viscous forces of the stationary flow balance the volume force generated by the high-frequency wave.