Electronic Supplementary Information for:
Dynamics and polarization of superparamagnetic chiral nanomotors in a rotating magnetic field

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I. Rotation matrix

We use the definition of the three Euler angles $\varphi$, $\theta$ and $\psi$ following Ref. [1]. The components of any vector $\mathbf{W}$ in the body-fixed coordinate system (BCS) and in the laboratory coordinate system (LCS) are determined from the relation $\mathbf{W}^{BCS} = \mathbf{R} \cdot \mathbf{W}$, where $\mathbf{R}$ is the rotation matrix. The rotation matrix is expressed explicitly via the Euler angles [2]

$$\mathbf{R} = \begin{pmatrix}
    c_\varphi c_\psi & s_\varphi s_\psi c_\theta & s_\varphi c_\psi c_\theta & s_\psi s_\theta \\
    -c_\varphi s_\psi & s_\varphi c_\psi c_\theta & -s_\varphi c_\psi & c_\varphi c_\psi c_\theta & c_\psi s_\theta \\
    s_\varphi s_\theta & -c_\varphi s_\theta & c_\theta & c_\varphi c_\theta & c_\psi s_\theta
\end{pmatrix},$$

where we use the compact notation, $s_\psi = \sin \psi$, $c_\theta = \cos \theta$, etc.

II. Approximate rotational viscous resistance coefficients of a helix

We approximate the rotational viscous resistance coefficients of a helical propeller by the corresponding values for a prolate spheroid approximating the helix. Let $a$ and $b$ be, correspondingly, the longitudinal (along the symmetry axis) and transversal semi-axes of the spheroid. The respective viscous resistances due to rotation about the symmetry axis and in perpendicular direction read [3]

$$\kappa_\parallel = 2\eta V n_\parallel^{-1}, \quad \kappa_\perp = 2\eta V \frac{a^2 + b^2}{a^2 n_\parallel + b^2 n_\perp},$$

(S1)

where $\eta$ is the dynamic viscosity of the liquid, $V$ is the spheroid volume, $n_\parallel$ and $n_\perp = (1 - n_\parallel)/2$ are the depolarizing factors of the spheroid. For the prolate spheroid with $a > b$ and eccentricity $e = \sqrt{1 - b^2/a^2}$ the depolarizing factor along the symmetry axis reads [4]

$$n_\parallel = \frac{1 - e^2}{e^3} \left( \frac{1}{2} \ln \frac{1 + e}{1 - e} - e \right).$$

(S2)

III. Particle-based numerical algorithm

The numerical procedure used to compute the various components of the viscous resistance tensor is based on multipole expansion scheme [5]. The filament is constructed from nearly touching $N$ rigid spheres (“shish-kebab” filament) having the same radius $r = 1$. The no-slip condition at the surface of all spheres is enforced rigorously via the use of direct
transformation between solid spherical harmonics centered at origins of different spheres. The method yields a system of $O(N L^2)$ linear equations for the expansion coefficients and the accuracy of calculations is controlled by the number of spherical harmonics (i.e. truncation level), $L$, retained in the series. This approach has been applied before for modeling low-Reynolds-number swimmers, e.g., rotating helix [6, 7] and undulating filament [8].

The spheres composing the helical filament are partitioned along the backbone of the filament $X(s)$ (see Eq. (21) and Fig. S1) so that the distance between centers of neighboring spheres is set to $2.02r$. The motion of the $i$th sphere composing a helix can be decomposed into translation $U_i$ and rotation $\omega_i$ about its center, as $V_i = U_i + \omega_i \times r_i$ with $r_i$ being the radius vector with origin at the center of $i$th sphere. For any prescribed rigid-body-motion of the helix, $\{U_i, \omega_i\}$ are determined uniquely. For instance, for computing the components of the resistance tensor, such as $\xi_{\parallel}, \kappa_{\parallel}$ and $B_{\parallel}$, associated with translation $U$ and rotation $\omega$ about the $x_3$-axis, one has $\omega_i = e_3\omega$ and $U_i = U e_3 + \omega e_3 \times R_i$, where $R_i$ is a position vector to the $i$th sphere center.

FIG. S1. Illustration of particle-based “shish-kebab” 2-turn helix approximating the regular helix with circular cross section of radius $r = 1$ (transparent blue) with helical radius $R = 2.5$ and pitch angle $\Theta = 65^\circ$. 

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IV. Demagnetizing factors of infinitely long elliptic cylinder

The demagnetizing factor $N$ of infinitely long cylinder with an elliptic cross-section with corresponding semi-axes $\hat{a}$ and $\hat{b}$ was reported in Ref. [9]:

$$N = (2\pi)^{-1} \left[ 4 \arctan \frac{\hat{a}}{\hat{b}} + \frac{2\hat{b}}{\hat{a}} \ln \frac{\hat{b}}{\hat{a}} + \left( \frac{\hat{a}}{\hat{b}} - \frac{\hat{b}}{\hat{a}} \right) \ln \left( 1 + \frac{\hat{b}^2}{\hat{a}^2} \right) \right].$$

At $\hat{a} > \hat{b}$ it determines the demagnetization factor $N_1$ along the short axis. The demagnetizing factor $N_2$ can be found either by the permutation $\hat{a} \leftrightarrow \hat{b}$ or from the equality $N_1 + N_2 = 1$.

For the regular helix with circular cross-section $N_1 = N_2 = 1/2$.