Supplementary Data 1: Mathematical equations used for modeling

(1) Umklapp phonon-phonon scattering events\cite{1}:

\[
\tau_{\text{Umklapp}}^{-1} = 2\gamma^2 k_B T \omega^4 
\]

where $\gamma$ is Gruneisen anharmonicity parameter, $\mu$ is shear modulus, $V_0$ is volume per atom, $\omega_D$ is Debye frequency, $T$ is ambient temperature and $\omega$ is phonon angular frequency.

(2) mass-difference impurity scattering\cite{2}:

\[
\tau_{\text{defect}}^{-1} = \frac{nV^2\omega^4(\Delta M)^2}{4\pi v_s^3 M}
\]

where $v_s$ is an averaged phonon group velocity, $n$ is the dopant concentration is denoted, $V$ is the volume of the host (silicon) atom, $M$ is the silicon atomic mass, and $\Delta M$ is the difference between the host and impurity atomic masses.

(3) Phonon to grain boundary scattering\cite{3}:

\[
\tau_{\text{grain}}^{-1} = \int_0^d \frac{v_s}{d_g(z)} \left( \frac{1-p_{\text{tr}}}{1+p_{\text{tr}}} \right) dz
\]

where $d_g(z)$ is the grain size at thickness $z$, $p_{\text{tr}}$ is the probability of specular transmission ($0 \leq p_{\text{tr}} \leq 1$).

(4) Segregated dopants induced scattering\cite{4}:

\[
\tau_{\text{segregated dopants}}^{-1} = \int_0^d \frac{2v_s}{\pi d_g(z)} \left( 1 + \left[ e^{\left( \frac{[n]^2}{2} \delta(\omega,z) \right)} - 1 \right]^{-1} \right)^{-1} dz
\]

where $\delta(\omega,z)$ is the grain boundary scattering strength.

Table S1 The parameters used for simulation

<table>
<thead>
<tr>
<th>Temperature (K)</th>
<th>Doping concentration (cm$^{-3}$)</th>
<th>Maximum grain size (nm)</th>
<th>Minimum grain size (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>$3 \times 10^{19}$</td>
<td>400</td>
<td>100</td>
</tr>
</tbody>
</table>
Supplementary Data 2: Measurement setup of thermal electric properties of poly-Si

Figure S1 Testing setup for the measurement of thermoelectric properties of poly-Si.

The measurement is conducted with a probe-station (Cascade Microtech, PMV200) equipped with a vacuum chamber, which can provide vacuum level at 10^{-3} mbar, to eliminate the influence of the thermal conductance due to air. The Chip is clamped onto a metal chuck, which acts as a large heat sink. The temperature of the chuck is controlled by a temperature stabilizer to maintain a constant temperature. The output signal can be readout by the probe and received by a semiconductor parameter analyzer (Agilent technology, 4156C)
Supplementary Data 3: Characterization results of the test key

Figure S2 The characterization with input power vs. output voltage of n-type poly-Si (a) and p-type poly-Si (b) test-key at room temperature.

As shown in this figure, the output is quite linear to the input power which suggests the contact between the Al and poly-Si is quite good and the temperature rise caused by the micro-heater do not influence the performance of thermocouple obviously.

