Supplement Information

Broadband Slow Light in One – Dimensional Logically Combined Photonic Crystals

G. Alagappan* and C. E. Png

Department of Electronics and Photonics, Institute of High Performance Computing, Agency for Science, Technology, and Research (A-STAR), Fusionopolis, 1 Fusionopolis Way, #16-16 Connexis, Singapore 138632

*Corresponding author: gandhi@ihpc.a-star.edu.sg
**Bloch mode decomposition technique**

Here, we will describe the details of the Bloch mode decomposition technique. This technique is used to calculate the Bloch mode expansion coefficient, $a_n$. [see Figure 2(e)]. The modes of PC1 are Bloch modes. Each of the Bloch mode can be identified with a pair of indices that denote the band index, $n$, and the wavevector, $k$. The Bloch mode, $\phi_{n,k}(x)$, of PC1 with frequency $\omega_{n,k}$, satisfy the 1D time – independent Maxwell’s equation,

$$\left[\frac{d^2}{dx^2} + \frac{\omega_{n,k}^2}{c^2} \epsilon_i(x)\right] \phi_{n,k}(x) = 0 \quad [S1],$$

where $\epsilon_i(x)$ is the dielectric function of PC1. The Bloch modes of PC1 obey the orthogonalization condition,

$$\int_{-a/2}^{a/2} \phi_{m,k}^*(x) \phi_{n,k}(x) dx = \delta_{mm} \delta_{kk}. \quad [S2].$$

Let us expand the electric field, $E(x)$, in the LCPC using the complete set of PC1’s Bloch modes:

$$E(x) = \sum_{n,k} a_{n,k} \phi_{n,k}(x) \quad [S3].$$

This electric field obeys the 1D time – independent Maxwell’s equation,

$$\left[\frac{d^2}{dx^2} + \frac{\omega^2}{c^2} \epsilon(x)\right] E(x) = 0 \quad [S4],$$

where $\epsilon(x)$ is the dielectric function of the LCPC. Expressing $\epsilon(x) = \epsilon_i(x) + \epsilon_p(x)$, and using Eqns. S1 – S3, we can transform Eqn. S4, into a system of linear equations,
\[ \frac{1}{a_s} \sum_{n,k} a_{n,k} \int_{-a_s/2}^{a_s/2} \phi_{m,k}^* (x) e_p (x) \phi_{n,k} (x) dx = \left( \frac{\omega_{m,k}^2}{\omega^2} - 1 \right) a_{m,k}. \quad [S5]. \]

Writing \( \frac{1}{a_s} \int_{-a_s/2}^{a_s/2} \phi_{m,k}^* (x) e_p (x) \phi_{n,k} (x) dx \) as \( \langle \phi_{m,k}^* | e_p | \phi_{n,k} \rangle \), and re-arranging Eqn. S5, we arrive at the following symmetrical eigenvalue problem,

\[ \sum_{n,k} \langle \phi_{m,k}^* | e_p | \phi_{n,k} \rangle + \delta_{m,n,k',m'} [a_{n,k} \omega_{n,k}] = \frac{c^2}{\omega^2} a_{m,k} \omega_{m,k}. \quad [S6], \]

with the eigenvalues \( c^2 / \omega^2 \).

Before proceeding, let us examine the term, \( \langle \phi_{m,k}^* | e_p | \phi_{n,k} \rangle \), in Eqn. S6. This term describes the coupling of PC1’s Bloch mode due to the perturbation, \( e_p (x) \). Since, the period of \( e_p (x) \) is equal to \( a_s = Ra \), the conservation of the translational symmetry requires \( k - k' \) to be multiples of \( g = 2\pi/a_s = 2\pi/Ra \). This means, different Bloch modes of PC1 will couple to each other, only if their wavevectors differ by multiples of \( g \). [i.e., \( \langle \phi_{m,k}^* | e_p | \phi_{n,k} \rangle \neq 0 \), only when \( k - k' \) is a multiple of \( g \)]. At such, it is useful to consider a folded band structure of PC1 [see the main text for the details]. In the folded band structure, the wavevectors of the adjacent bands differ exactly by \( g \).

For a given wavevector, \( k \), within the first BZ of the LCPC, the Bloch modes in the same folded cannot couple to each other. Only modes with the same \( k \), but in different folded bands can couple. Therefore, for a given \( k \), we can re-write Eqn. S6 using a single index subscript,

\[ \sum_{n} \langle \phi_{m,k}^* | e_p | \phi_{n} \rangle + \delta_{m,n} [a_{n} \omega_{n}] = \frac{c^2}{\omega^2} a_{m} \omega_{m}. \quad [S7]. \]
Here, \( n \) is the index of the folded band. Eqn. S7 can be written in a matrix form as
\[
\hat{B} \mathbf{v} = \left[ c^2 / \omega^2 \right] \mathbf{v},
\]
where \( \mathbf{v} = [a, a_1, a_2, \ldots] \). Inverting this matrix equation we have,
\[
\hat{B}^{-1} \mathbf{v} = \left[ \omega^2 / c^2 \right] \mathbf{v}
\]
[S8].

Solving the eigenvalue problem in Eqn. S8, the expansion coefficients \( [a_n] \) can be found. One can also use Eqn. 8 to obtain the band structure of the LCPC. Please take note that, if we express both \( \phi_n \) and \( \varepsilon_p \) in the plane wave basis (i.e., using a Fourier series), then Eqn. S8 will revert to the standard equations that describes the well-known plane wave expansion technique [17]. However, the information on \( a_n \) will be lost in the plane wave basis.