Supplementary Material (ESI) for Soft Matter

Supplementary Information for

MODULATION OF ATTRACTIVE COLLOIDAL INTERACTIONS BY LIPID MEMBRANE FUNCTIONALIZATION

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Derivation of the relation between *U*(*r*) and observed single particle and pair position distributions

The pair interaction potential, $U(r)$, can be determined by measurement of both $p_{obs}(r)$, the observed distribution of separations, and $p_0(x)$, the distribution of positions, x, of a single particle trapped in the line.

Denoting the confining potential of the trap as $U_{ext}(x)$, this potential and $p_0(x)$ are related by a Boltzmann relation: $p_0(x) \propto \exp\left(-U_{ext}(x)/k_BT\right)$. Consider a line trap of length L centered at $x = 0$ with two trapped, interacting particles with position distribution functions $p_1(x_1)$ and $p_2(x_2)$. The probability distribution of their separations is

$$[Eq.\ S1] \quad p_{obs}(r) = A\int_{-L/2}^{L/2-r} p_1(x_1)\, p_2(x_1 | x_1 + r)\, dx_1 \ ,$$

where the first factor indicates the probability of finding particle 1 at $x = x_1$ and the second indicates the probability of finding particle 2 at $x = x_2$ given the position of particle 1, and A is a normalization constant. Particle 2 experiences a total energy $U(x_1, x_2) = U_{ext}(x_2) + U(r)$, where $r = x_2 - x_1$. Therefore

$$[Eq.\ S2] \quad p_2(x_2) = \exp\left(-\frac{U(x_1,x_2)}{k_BT}\right) = \exp\left(-\frac{U_{ext}(x_2)}{k_BT}\right)\exp\left(-\frac{U(r)}{k_BT}\right) = p_0(x)p(r) \ ,$$

neglecting normalization factors, where $p(r)$ is the "true"(not observed) distribution function for the pair separations. Combining Eqs. S1 and S2:

$$[Eq.\ S3] \quad p_{obs}(r) = A\int_{-L/2}^{L/2-r} p_0(x_1)\, p_0(x_1 + r)\, p(r)\, dx_1 = Ap(r)\int_{-L/2}^{L/2-r} p_0(x_1)\, p_0(x_1 + r)\, dx_1 \ .$$

Therefore, $p(r)$ is related to the observed $p_{obs}(r)$ by:

$$[Eq.\ S4] \quad p(r) = \frac{p_{obs}(r)}{\int_{-L/2}^{L/2-r} p_0(x)p_0(x+r)dx} \ ,$$

which gives $U(r)$ via the Boltzmann relation

$$[Eq.\ S5] \quad U(r) = -k_BT \ln p(r) .$$