

## Supporting Information for

### Mechanical Self-Assembly Fabrication of Microgears

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This supporting information includes:

- I. Computational method
- II. Theory: Critical buckling wave number
- III. Theory: Buckling amplitude
- IV. More details of experimental method
- V. Varying substrate modulus with mixing ratios

#### I. Computational method

Buckling simulation is carried out using finite element method (FEM) with software ABAQUS. The substrate is meshed by over 100,000 three-dimensional hexahedron elements, and the film is represented by over 10,000 four-node general purpose shell elements with reduce integration and accounting for large rotation. The number of elements is determined through a mesh convergence study.

#### II. Theory: Critical buckling wave number

The critical buckling wave number can be obtained through the linear perturbation stability analysis. The normal force  $N$  and bending moment  $M$  in the theoretical plane strain model in the text are defined as  $N = \bar{E}_f A \varepsilon$  and  $M = \bar{E}_f I \kappa$ , respectively. Here,  $A$  and  $I$  are the area and moment of inertia of the cross-section of the film. The stretching strain and bending curvature variation in the text are defined as  $\varepsilon = (v_{,\theta} + w)/R + (w_{,\theta} - v)^2/2R^2$  and  $\kappa = (w_{,\theta\theta} - v_{,\theta})/R^2$ , respectively, where  $(\cdot)_{,\theta} = d(\cdot)/d\theta$  defines the derivative with respect to the circumferential angle  $\theta$ .  $v$  is the tangential displacement of the film. The buckling perturbation mode is assumed to take a sinusoidal form, i.e.  $v = \hat{v} \sin n\theta$  and  $w = \hat{w} \cos n\theta$ .  $\hat{v}$  and  $\hat{w}$  are the amplitude of the buckling mode and  $n$  is the circumferential wave number. The system potential energy can be further expressed as a function of  $\hat{v}$ ,  $\hat{w}$  and  $\theta$  by

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substituting the assumed buckling mode into related expressions. Through a minimization of the total energy with respect to  $\hat{v}$  and  $\hat{w}$ , both the critical buckling load and the corresponding critical buckling wave number can be deduced.

### III. Theory: Buckling amplitude

The post-buckling analysis is highly nonlinear and can not be obtained from the above linearization method. The amplitude of the gear teeth can be derived from the deformation compatibility at the interface. In the buckled state (with subscript 1), the normal and tangential film displacements are assumed to remain to be  $w_1 = \hat{w} \cos n\theta$  and  $v_1 = \hat{v} \sin n\theta$ . Since the film is assumed to remain bonded to the substrate during the post-buckling process, an additional constraint must be imposed to ensure the overall film displacement matches that of the underlying substrate. This constraint condition can be expressed as

$$\frac{\sigma_1 - \sigma_0}{\bar{E}_f} = \frac{1}{2\pi R} \int_0^{2\pi} \left[ \frac{v_{1,\theta} + w_1}{R} + \frac{1}{2} \left( \frac{w_{1,\theta}}{R} \right)^2 \right] R d\theta \quad (s1)$$

where  $\sigma_1$  and  $\sigma_0$  are the film stress in the pre-buckling and post-buckling states, respectively. After omission of the first order terms in Eq. (s1) via the periodicity of deformation, the amplitude of the buckling mode is obtained:

$$\frac{\hat{w}}{t} = \frac{R}{t} \frac{2}{n} \left( \frac{\sigma_1 - \sigma_0}{\bar{E}_f} \right)^{\frac{1}{2}} \quad (s2)$$

For the critical mode with  $n_{cr} = R \left[ K / (\bar{E}_f I) \right]^{\frac{1}{4}}$  and  $\sigma_0 = \sigma_{cr}$  given by Eq.(2) and Eq. (3), the amplitude in Eq. (4) in the text can be deduced.

### IV. More details of experimental method

To validate the concept of mechanical self-assembly on a curved substrate, a cylindrical film/substrate system was prepared via a two-step casting process. Several molds were first created using ultra high molecular weight polyethylene (UHMWPE) with varying circular hole diameters (0.5in, 0.375in, 0.3125in and 0.25 in). The soft cylindrical substrate was prepared by the solidification of a gel made of polyurethane solution. A polyurethane compound of Hydrospon™ 400 (Industrial Polymers Corporation, Houston, TX) was thoroughly mixed with distilled water at a certain mixing ratio for 15 seconds at room temperature, and poured into the mold. Both  $L$  and  $R$  of the substrate can be controlled quite precisely. The mixing ratios are varied at 1:1, 1.5:1, and 2:1 in this study to control the substrate modulus, see below. A thin circular polyvinyl chloride (PVC) film (of 50  $\mu m$  thickness) with an extremely thin polychloroprene adhesive layer was immediately adhered to the lateral surface of the cylindrical substrate and remains bonded to the substrate throughout subsequent processes. The cylindrical film/substrate system was removed from the mold and hang in a desiccator chamber to cure (for up to two days), and the buckle profiles are continuously recorded.

## **V. Varying substrate modulus with mixing ratios**

The substrate material modulus used in experiment is varying with mixing ratio. A uniaxial tensile test (using an Instron 4400 machine) was performed to determine the modulus at different mixing ratios (1:1, 1.5:1, and 2:1, measured immediately after mixing) and the film. The Young's modulus of the substrate almost decreases linearly with the mixing ratios, with specific numbers reported in the text.