Solid-supported thin elastomer films
deformed by microdrops

Ramon Pericet-Camara*, Günter K. Auernhammer, Kaloian Koynov, Simone Lorenzoni,
Roberto Raiteri, Elmar Bonaccurso*

Supplementary Information

In the work of Long et al.¹, the existence of a microtrough close to the ridge at the TPCL is explained by the combination of two limiting cases. Analytic approximations can be given for distortions of the surface with a wave number \( q \) much smaller than the film thickness \( t \) (\( q \cdot t \ll 1 \)) and with very high wave numbers (\( q \cdot t \gg 1 \)). To get a more intuitive impression for the mechanisms involved, it is useful to consider the Fourier transformation of the deformation. Frederickson et al.² did calculate the energy density of a deformation \( h(x) \) of a thin layer of an elastic material to be

\[
F[h] = \frac{1}{2} \int \left( \frac{d^2 q}{(2\pi)^3} P(q) \cdot h(q) \cdot h(-q) \right), \tag{4}
\]

where \( h(q) \) is the Fourier transform of the surface deformation \( h(x) \) and \( P(q) \) is the energy contribution in the wave vector interval from \( q \) to \( q+dq \). In simplified form, \( P(q) \) is given by

\[
P(q) = \frac{1}{l^2} \left[ \frac{\gamma}{(q \cdot t)^2} + E \cdot t \cdot \frac{3}{(q \cdot t)^2 + 2 \cdot q \cdot t} \right]. \tag{5}
\]

As depicted in the main part of Fig. S1, this function shows a pronounced minimum for \( q \) in the order unity. Both short and long wave length deformations cost more energy than deformations with wavelength of the order of the layer thickness. As already stated by Frederickson et al., the actual surface profile must minimize the free energy functional (4). Starting close to the TPCL with a monotonic profile of the ridge (e.g. the logarithmic decay predicted by Rusanov), the system can reduce its energy by reducing the amplitude of the Fourier modes where \( P(q) \) takes large values and by increasing the amplitude of energetically less expensive modes, i.e., modes close to the minimum of \( P(q) \). To illustrate this, we use a test function that combines both the logarithmic decay
(close to the TPCL) and an exponentially decreasing modulation at higher distances. The parameter $b$ scales the amplitude of the oscillatory part of the profile.

$$h(x) \sim \frac{1 + b \cdot \cos(\frac{x}{\xi_1})}{1 + b} \ln(|x| \cdot \exp(-\frac{|x|}{\xi_2})]. \tag{6}$$

The inset to Fig. S1 gives the Fourier transform of (6) in two cases. The dashed line shows the Fourier transform of a monotonically decaying profile ($b = 0$) and the solid line gives the Fourier transform of a profile with a microtrough at approximately the layer thickness ($b \gg 1$). The free energy as calculated by (4) is smaller for the profile with the microtrough. Since the mechanism of this energy reduction is actually not dependent on the details of $h(q)$ but a rather general feature of $P(q)$, similar results could be reproduced with various test functions.

To summarize, the formation of the microtrough reduces the contribution of the low wave vector (long wavelength) part in $h(q)$ on the expense of intermediate wave vectors and globally reduces the energy stored in the surface deformation.

![Graph showing the energy density](image)

**Figure S1:** Qualitative behavior of the energy density in the interval from $q$ to $q+dq$ as deduced by Frederickson et al. The inset shows the Fourier transform of a profile with (solid line, as observed in our experiments) and without (dashed line) microtrough close to the TPCL.
