

## Methods

### WLM Preparation

This study uses the surfactant cetyltrimethylammonium bromide (CTAB, cetyl = hexadecyl, Alfa Aesar) dissolved in  ${}^2\text{H}_2\text{O}$  (Cambridge Isotope Laboratories, 99.9%  ${}^2\text{H}$ ) and regulated at 45 °C with gentle stirring for 48 h. CTAB concentrations of 18.0, 19.0, 20.0, 21.0, 22.0, and 23.0 wt % ( $\pm 0.1$  wt %) of total solution (0.67, 0.71, 0.76, 0.81, 0.86, and 0.91 M, respectively) are used for rheo-NMR studies. At least 24 h prior to experiments, samples are poured into a Couette cell, sealed with Parafilm, and allowed to rest at 40 °C. This ensures that measurements are not biased by shear history during cell assembly. Solutions are maintained above 25°C to prevent CTAB crystallization.

### 15 Rheo-NMR Spectroscopy

A Varian Unity NMR spectrometer ( $B_0 = 9.4$  T,  $v_{2\text{H}} = 61.395$  MHz) applies a pulse-acquire sequence with 90° pulse of 25.0  $\mu\text{s}$ .  ${}^2\text{H}$  spectra are single acquisitions in order to accurately map sample behavior over time with  $\leq 1$  s time resolution. Typical FWHM linewidths in the  ${}^2\text{H}$  spectra are  $\sim 2$  Hz (shim limited). The custom rheo-NMR apparatus consists of a Couette cell (concentric cylinders) with rotation axis along  $B_0$ , comprised of a 10 mm O.D. NMR tube (9.03 mm I.D.) and a 7.45 mm O.D. NMR tube<sup>31</sup> separated by Delrin® spacers. CTAB sample is held in the 0.79 mm gap. This cell produces  $\dot{\gamma} = 31.4 \pm 0.5 \text{ s}^{-1}$  per 1 Hz revolution of the inner cylinder.<sup>31</sup> The length of the Couette cell is  $\sim 60$  mm, ensuring that sample fills the RF coil. Shear rate ( $\dot{\gamma}$ ) inside this cell is given by:

$$\dot{\gamma}^\bullet = v_I \left( \frac{D_1 \pi}{D_2 - D_1} \right) \quad (2)$$

where  $D_1$  is the outer diameter of the inner cylinder,  $D_2$  is the inner diameter of the outer cylinder, and  $v_I$  is the rotation frequency of the inner cylinder. Douglass *et al.*<sup>31</sup> have estimated the maximum practical shear rate at  $\approx 200 \text{ s}^{-1}$ . Shear rate is varied using a computer interface (software: SMC60WIN SD130, v. 2.01; programmable driver: DPY50601, Anaheim Automation), which controls a stepper motor mounted above the NMR magnet. A brass drive shaft rotating on Teflon® bearings couples the stepper motor to the inner cylinder of the Couette cell. The cell's outer cylinder is fixed to the probe by a rubber annulus. Interchangeable stepper motors with varying gear ratios (1:1 motor #23YSG106S-LW8, 1:12.5 motor #23YSG106S-LW8-R12.5, and 1:100 motor #23YSG106S-LW8-R100, Anaheim Automation) allow access to over five decades of shear rate.