**Motion Induced by Asymmetric Enzymatic Degradation of Hydrogels**

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**Supplementary Material**

**S1. Degradation-induced motion of beads**

**Supplementary Movie 1:** Motion of beads, gelatin, and trypsin in a gradient of trypsin concentration. The data was acquired at 12 frames per minute. The movie plays at 360x speed up. At time = 0, the right edge of the gelatin was exposed to a solution of 0.5 mg/mL trypsin in digested gelatin. Trypsin diffused into and degraded the gelatin, and beads moved up the trypsin gradient. (Top panel) Trans-illuminated image of beads. (Middle panel) Green fluorescence of gelatin labeled with AF488. (Bottom panel) Red fluorescence of trypsin labeled with AF647. Beads are 6 µm diameter, embedded in 5% gelatin.

**S2. Transport of enzyme through gel**

If the transport of the enzyme is purely diffusive and our sample is translationally invariant in \(y\), then the concentration profile of enzyme, \(c(x, t)\) is governed by the 1-D diffusion equation:

\[
\frac{\partial c(x,t)}{\partial t} = D \frac{\partial^2 c(x,t)}{\partial x^2}
\]  

(S1)

where \(D\) is the diffusion coefficient of enzyme through the gel. If the gel resides in a semi-infinite plane \(x \geq 0\), the solution to Eq. S1 with the initial condition \(c(x,0) = c_0 \theta(-x)\) yields the following functional form for the dimensionless mean position of the enzyme, \(\langle \bar{x}_{enz}(t) \rangle\), over a region \(x \in [0,x_f]\):

\[
\langle \bar{x}_{enz}(t) \rangle = \frac{Dt}{2x_f} \text{erf}\left(\frac{x_f}{2\sqrt{Dt}}\right) + \frac{1}{2} x_f \left(-2 \sqrt{\frac{D}{\pi}} \text{exp}\left(-\frac{x_f^2}{4Dt}\right) + x_f \text{erfc}\left(\frac{x_f}{2\sqrt{Dt}}\right)\right)
\]

(S2)

\[
\bar{x}_{enz}(t) = \frac{2}{\sqrt{\pi}} \int_0^t \text{e}^{-\frac{t'}{4Dt}} \text{d}t' + x_f \text{erfc}\left(\frac{x_f}{2\sqrt{Dt}}\right)
\]

where \(\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} \text{d}t\) and \(\text{erfc}(x) = 1 - \text{erf}(x)\).