Electronic Supplementary Material (ESI) for Soft Matter

Supplementary material

Elasto-capillary meniscus : Pulling out a soft strip sticking to a liquid surface

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1 Experimental setup

Figure 1 shows a photograph of the experimental setup. Note that the water has been dyed for the need of visualization. Both the syringe used for drainage and the transversal length of the cell have been made visible on the picture.

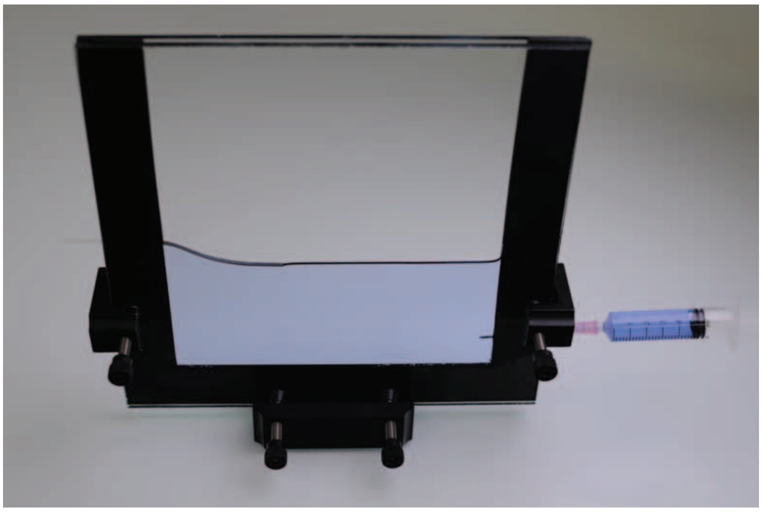

Figure 1 Experimental setup.

2 Derivation of the leading system of equations

Here we detail how we derive the leading equation appearing in the main text. Starting from Kirchhoff equation along Z direction, which is:

$$B\theta''(S) = F_x(S)\sin\theta(S) - F_y(S)\cos\theta(S) \tag{1}$$

we derive it once with respect to S :

$$B\,\theta'''(S) = F_x'(S)\sin\theta(S) - F_y'(S)\cos\theta(S) + \theta'(S)\,(F_x(S)\cos\theta(S) + F_y(S)\sin\theta(S)) \quad (2)$$

Introducing now in the last expression Kirchhoff equations along X and Y, which are:

$$F_x'(S) = -\rho g\,(Y(S) + H)\sin\theta(S) \quad (3)$$
$$F_y'(S) = \rho g\,(Y(S) + H)\cos\theta(S) \ , \quad (4)$$

we obtain:

$$B\,\theta'''(S) = -\rho g(y(S) + H) + \theta'(S)g(S) \ , \quad (5)$$

where we put:

$$g(S) = F_x(S)\cos\theta(S) + F_y(S)\sin\theta(S) \ . \quad (6)$$

We now derive $g(S)$ with respect to S:

$$g'(S) = F_x'(S)\cos\theta(S) + F_y'(S)\sin\theta(S) + \theta'(S)\,(-F_x(S)\sin\theta(S) + F_y\cos\theta(S)) \quad (7)$$

We introduce again equations (3) and (4) to replace the first two terms, and equation (1) for the last one. We find:

$$g'(S) = -B\theta'(S)\theta''(S) \quad (8)$$
$$\Rightarrow \quad g(S) = B\left(-\frac{1}{2}\theta'(S)^2 + K\right) \quad (9)$$

We now have to determine the constant K. We first evaluate $g(S)$ at $S = L$:

$$g(L) = F_x(L)\cos\theta(L) + F_y(L)\sin\theta(L) = B(-\frac{1}{2}\theta'(L) + K) \quad (10)$$

Then, we introduce the boundary conditions at $S = L$, detailed in the main text, which are:

$$\theta'(L) = 0 \quad ; \quad F_x(L) = -\gamma\cos(\theta(L) + \varphi) \quad ; \quad F_y(L) = -\gamma\sin(\theta(L) + \varphi) \quad (11)$$

and we obtain:

$$g(L) = -\gamma\cos\varphi = BK \quad (12)$$

Therefore, we can finally write the leading equation which appears in the main text:

$$B\,\theta'''(S) = -\rho g(y(S) + H) - B\frac{1}{2}\theta'(S)^3 - \gamma\theta'(S)\cos\varphi \quad (13)$$

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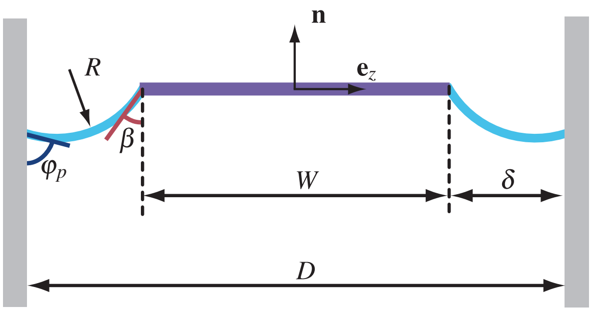

Figure 2 Schematic view of the elastocapillary meniscus in the $(\mathbf{n}, \mathbf{e}_z)$ plane, where $\mathbf{n}$ is the unitary vector normal to the strip. The gap between the strip and the glass walls is δ, and the liquid-air interface at this gap has a circular shape with radius R. β and φ_p designate respectively the angle between the direction of surface tension along the strip and the vertical direction, and the contact angle between the liquid and the glass walls.

3 Three-dimensional considerations

Here we present some three-dimensional aspects of the experimental setup.

3D correction of the leading equation. In the experimental setup a small gap exists between the strip and the lateral glass walls, in order to avoid friction. Therefore, a narrow liquid-air interface lies between the strip and the walls. This interface is subjected to the pressure jump which exists between the air, whose pressure is constant and equal to p_a, and the liquid, whose pressure is $p(Y) = p_a - \rho g(Y+H)$. Following Laplace law, the lateral interface has to be curved, and we refer to it as a lateral meniscus.

Figure 2 shows a 2D view in the $(\mathbf{n}, \mathbf{e}_z)$ plane. On the one hand, the experimental gap is $\delta \simeq 0.5\,\text{mm}$ and we neglect the effect of gravity on the shape of the lateral meniscus, since $\delta < L_{gc}$. On the other hand, the curvature of the meniscus in the $(\mathbf{n}, \mathbf{e}_z)$ plane, which scales as $1/\delta$, is much larger than the curvature in the (x, y) plane, which scales as $1/L$. For these two reasons, we consider the lateral meniscus as a circular arc situated in the $(\mathbf{n}, \mathbf{e}_z)$ plane. The radius R of the arc is dictated by Laplace law:

$$R = \frac{1}{\kappa} = \frac{\gamma}{\Delta p} = \frac{\gamma}{\rho g(Y+H)} = \frac{L_{gc}^2}{Y+H} \,. \tag{14}$$

Hence, the smallest radius is situated near the clamp, at $Y = 0$.

Due to the presence of this meniscus, surface tension is pulling the lateral edges of the strip. The vertical component of this distributed force is $\gamma \cos\beta$, with β the angle between the tangent to the lateral meniscus and the vertical direction (see Fig. 2). The value of β can be predicted following a simple cartesian scheme. One has to construct in the $(\mathbf{n}, \mathbf{e}_z)$ plane an arc of circle whose radius $R(Y)$ is given by equation (14); the arc of circle has to pass through the point $(Y, Z = W/2)$ and has to touch the line $Z = D/2$ with a given angle φ_Y. Here, $\varphi_Y \simeq 20°$ is the contact angle between the liquid and the glass walls. This cartesian problem can be solved and yields:

$$\beta(Y) = \frac{\pi}{2} - \arctan\left(\frac{-\delta + R\cos\varphi_Y}{\sqrt{R^2\sin^2\varphi_Y - \delta^2 + 2\delta R\cos\varphi_Y}}\right) \tag{15}$$

Depending on $R(Y)$, δ and φ_Y, $\cos\beta$ can be either negative or positive, meaning that surface tension is pulling the lateral edges upward or downward, respectively. It is now possible to introduce this 3D correction in Kirchhoff equations:

$$F_x'(S) = [-\rho g\,(Y(S) + H) - 2\widehat{\gamma}\cos\beta(S)]\sin\theta(S) \tag{16}$$

$$F_y'(S) = [\rho g\,(Y(S) + H) - 2\widehat{\gamma}\cos\beta(S)]\cos\theta(S) \tag{17}$$

where $\widehat{\gamma} = \gamma/W$, as these equations are written per unit width. Therefore, the leading equation can be corrected too, and it becomes:

$$\theta''' + (y+h) + \frac{1}{2}\theta'^3 - \ell_{\text{gc}}^2\theta'\cos\varphi - \frac{2\ell_{\text{gc}}^2}{w}\cos\beta = 0 \tag{18}$$

where $w = W/L_{\text{eh}}$.

Lateral failure of the elastocapillary meniscus. From a mathematical point of view, equation (15) contains a square root function, and for this reason it may happen that no real solution exists for β. In particular, this appears when R is too small compared to δ, and in the geometric problem it is related to the fact that the radius of the circular arc is too small to fit all the cartesian conditions described above. In the physical problem, this situation corresponds to the fact that the pressure jump imposed on the system, and in particular near the embedding, is too large compared to the admissible deformation of the lateral meniscus. Experimentally, we observe that the lateral meniscus detaches from the strip, causing air invasion below the strip from the lateral gaps. This situation causes the failure of the elastocapillary meniscus. A snapshot of this event is given in the next section (see figure 3).

4 Failure of the system : movies

We show in Fig. 3 of the manuscript a sequence of a failure of the elasto-capillary meniscus related to air invasion form the tip of the strip (see also supplementary movie). Another possibility for the collapse is illustrated Fig. 3, with a sequence of elasto-capillary meniscus failure due to air invasion from lateral gaps, as discussed in the previous section.

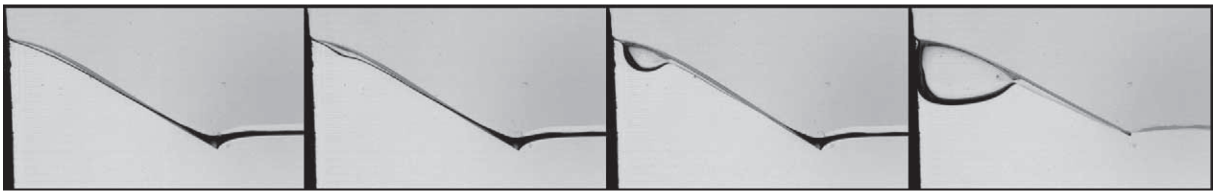

Figure 3 Sequence illustrating the failure of the elastocapillary meniscus because of air invasion from the lateral gaps. Intervals between two consecutive images are 239 ms, 48 ms and 59 ms.