

A: Animation for a single ring in dilute solution

The parameters for animations (the name of the file is RingsUnderShear60) are $N = 60$ and $Wi = 188$. The monomers are colored by red and green for illustration. Obviously, the animations present two distinct types of motion, tumbling (TB) and tank-treading (TT). When a ring polymer makes a TB motion, the chain undergoes a series of stretching, collapsing, tumbling, and restretching events. When the ring is stretched with an elliptical shape in the flow-gradient plane, another qualitative motion, TT motion, presents, where the monomers move along the contour. The animations also show that a motion can have characteristics of these two motions, and is denoted as TB-TT in the manuscript.

B: Animation for a single ring in linear polymer melts

Here, we show animations of a ring in linear polymer melts under shear flow. The simulation is performed by NEMD and Lees-Edwards boundary conditions are used to impose a shear flow. The parameters for animations are $\rho = 0.85$, $N_{ring} = 60$, $N_{linear} = 20$, and $\dot{\gamma} = 0.2$. The monomers of the ring are colored by red and green and the linear chains are colored by gray for illustration. Two files, SingleRingsInMeltsA and SingleRingsInMeltsB, tell the same story. For simplicity, we only show 1/10 linear polymers (200 chains) in SingleRingsInMeltsA. In order to observe the motions of the ring in SingleRingsInMeltsA clearly, we fix the center of mass of the ring at the center of the box and only exhibit the ring chain, as shown in SingleRingsInMeltsB. Obviously, the dynamical behavior of rings in melts is similar to those in solutions described in our manuscript.