Membrane elastic energy

We assume that the membrane elastic energy is minimized at zero curvature. The membrane elastic energy is then expressed by

\[ E_{\text{bound}} = \kappa_{\text{bound}} H_{\text{bound}}^2 A_{\text{bound}} \]  
(4)

\[ E_{\text{swell}} = \kappa_{\text{swell}} H_{\text{swell}}^2 A_{\text{swell}} \]  
(5)

where \( \kappa_i \) and \( A_i \) denote the bending rigidity and area, respectively, of a state \( i \), and \( H_i \) represents the mean curvature of the membrane of the state \( i \). We also assume energy storage around the hole, which is expressed by

\[ E_h = \frac{\lambda (H_{\text{bound}} + H_{\text{boundmax}})}{2\pi (r_p - r_{\text{swell}})} \]  
(6)

The volume of the buffer solution contained in a vesicle is approximately expressed as

\[ V_{\text{swell}} = \pi r_{\text{swell}}^2 \times 2\pi r_p = 2\pi^2 \left( \frac{1}{2H_{\text{swell}}} \right)^2 r_p \]  
(a1)

Parameters \( r_{\text{swell}} \) and \( r_p \) are shown in Figure 7a. Thus, \( E_{\text{swell}} \) may be rewritten as

\[ E_{\text{swell}} = \kappa_{\text{swell}} \frac{\pi^2 r_p}{2V_{\text{swell}}} A_{\text{swell}} \]  
(a2)

\( H_{\text{bound}} \) and \( r_p \) are dependent on each other by a geometric constraint:

\[ r_p = \left( \frac{A_{\text{bound}}}{\pi} \right)^{1/2} \left( 1 - \frac{A_{\text{bound}}}{4\pi H_{\text{bound}}^2} \right)^{1/2} \]  
(a3)

Here we use

\[ \sin \theta = \frac{r_p}{r_{\text{bound}}}, \quad A_{\text{bound}} = 2\pi r_{\text{bound}}^2 (1 - \cos \theta) \]

The first equation is an approximation. Using eq. a3, \( E_{\text{swell}} \) and \( E_h \) are expressed by

\[ E_{\text{swell}} = \kappa_{\text{swell}} \frac{\pi^{3/2} A_{\text{swell}}}{2 V_{\text{swell}}} \frac{A_{\text{bound}}}{4\pi} \left( 1 - \frac{A_{\text{bound}}}{4\pi H_{\text{bound}}^2} \right)^{1/2} \]  
(a4)
\[ E_h^{-1} = 2\pi \left( \lambda (H_{\text{bound}} + H_{\text{boundmax}}) \right)^{-1} r_p \left( 1 - \left( \frac{V_{\text{swell}}}{2\pi^2} \right)^{1/2} \left( \frac{A_{\text{bound}}}{\pi} \right)^{-3/4} \left( 1 - H_{\text{bound}}^* \right)^{-3/4} \right) \]  \tag{a5}

Here we introduce
\[ H_{\text{bound}}^* = \frac{A_{\text{bound}} H_{\text{bound}}^2}{4\pi}, \quad p_A^* = \frac{A_{\text{swell}} A_{\text{bound}}}{V_{\text{swell}}} \]  \tag{a6}

to obtain
\[ E^* = \frac{E_{\text{bound}} + E_{\text{swell}} + E_h}{2\pi \kappa_{\text{swell}}} \]
\[ = 2\kappa^* H_{\text{bound}}^* \frac{2 + \sqrt{\frac{\pi}{4}} p_A^* \left( 1 - H_{\text{bound}}^* \right)^{1/2}}{1 - \frac{1}{\pi^{1/4}} \left( 2r_A p_A^* \right)^{1/2} \left( 1 - H_{\text{bound}}^* \right)^{3/4}} \]  \tag{a7}

where \( E^* = (E_{\text{bound}} + E_{\text{swell}} + E_h) / (2\pi \kappa_{\text{swell}}) \), and dimensionless bending rigidity is defined as \( \kappa^* = \kappa_{\text{bound}} / \kappa_{\text{swell}} \).

\[ V_{\text{swell}} \] given by eq. a1 and is conserved. Eqs. a1 and a3 yield
\[ r_{\text{swell}} = \frac{1}{2H_{\text{swell}}} = \frac{V_{\text{swell}}^{1/2}}{\sqrt{2\pi}} \left( \frac{A_{\text{bound}}}{\pi} \right)^{-1/4} \left( 1 - \frac{A_{\text{bound}}}{4\pi} H_{\text{bound}}^2 \right)^{-1/4} \]  \tag{a8}

Using eqs. a3 and a8, we obtain
\[ A_{\text{swell}} = 2\pi r_{\text{swell}}^2 2\pi r_p = 2\sqrt{2\pi} V_{\text{swell}}^{1/2} \left( \frac{A_{\text{bound}}}{\pi} \right)^{1/4} \left( 1 - \frac{A_{\text{bound}}}{4\pi} H_{\text{bound}}^2 \right)^{1/4} \]  \tag{a9}

Eqs. a6 and a9 give
\[ p_A^* = 8\sqrt{\pi^3} r_A^2 \left( 1 - H_{\text{bound}}^* \right)^{3/2} \]  \tag{a10}

where \( r_A^* \) is defined as \( A_{\text{bound}} / A_{\text{swell}} \).

The substitution of eq. a10 for \( p_A^* \) in eq. a7 gives
\[ E^* = 2\kappa^* H_{\text{bound}}^* \frac{2 + 2\pi^2 r_p}{1 - H_{\text{bound}}^*} + \frac{\lambda^* H_{\text{boundmax}} + H_{\text{bound}}^*}{2}\left( 1 - H_{\text{bound}}^* \right)^{1/2} \left( 1 - \frac{1}{4\pi r_A^*} \frac{1}{1 - H_{\text{bound}}^*} \right) \]  \tag{a11}
which generates eq. 7 in the main text.

The total area of the membrane ($A_{\text{total}}$) is conserved. The equation $A_{\text{total}} = A_{\text{swell}} + 2A_{\text{bound}}$, eq. a9 and $r_A^* = A_{\text{bound}} / A_{\text{swell}}$ yield eqs. 9c and 9d in the main text. Here we used $\alpha_{\text{swell}}^* = 1 - 2\alpha_{\text{bound}}^*$, that is derived from $A_{\text{swell}} = A_{\text{total}} - 2A_{\text{bound}}$. 