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Measurement, accuracy and precision

Teachers' notes

Objectives

- Understand that data obtained during experiments are subject to uncertainty.
- Understand that the level of accuracy is linked to the context.
- Planning experiments and investigations.
- Making accurate observations.
- Evaluating data, considering anomalous results.

Outline

The teaching material is divided into three sections, all of which focus on an aspect of accuracy and precision. Each activity stands alone and is independent of the other two.

- Measuring uncertainties and reporting reliable results
- Choosing and using equipment
- Does being accurate really matter?

Teaching topics

The activities can be used at any point in a course to teach investigative skills and are suited to students in the 11–16 age range. The activities can be adapted easily to allow access to students of different ages and of different abilities. In the Measuring uncertainties and reporting reliable results section **The weighing experiment (method 2)** could be modified and used as a post-16 key skills exercise. Although the activities are quite general in nature, they can be used to teach specific skills. Alternatively, they could be used prior to carrying out investigations that require weighing out of materials, or the measurement of volume and temperature.

For example **Choosing and using equipment** and **Does being accurate really matter?** could be taught prior to or after carrying out experiments or investigations into:

- The neutralisation reaction between acid and alkali
- Rates of reaction
- Electrolysis
- Methods of separation.

Experimental details can be found in *Classic Chemistry Experiments*[1] Numbers 48; 29, 64, 65; 81, 82; 1, 4, 71 and 100 respectively.

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Section 1: Measuring uncertainties and reporting reliable results

Background information
Students (including post-16 students) are often confused about the meanings and difference between some of the vocabulary in regular use *eg*

- Accuracy and precision
- Repeatability and reproducibility
- Systematic error and true value
- Error and mistake
- Best fit line and anomalous points.

Definitions
Accurate – the result is close to a reference value or the average of the data is close to a reference value.

Precise – the data points are close together (but there can be a random error).

Repeatability – when the experiment is repeated by the same person, using the same equipment and the results are close together.

Reproducibility – when the experiment is carried out by different persons, using different equipment and the results are close together.

True value – a perfect value of the quantity, *eg* mass, volume, temperature. This is an ideal and can never be known exactly.

Reference value – A value taken to be very close to the true value and usually accepted as a point of reference, *eg* a 'standard weight' has been measured on a balance that has little or no error and so the 'measured weight' is very close to the true value and accepted.

Errors are not the same as mistakes *eg* not reading a scale correctly.

Systematic error – there is some problem with the apparatus, because the results are precise (close together), but not accurate.

Instrumental errors – *ie* quantifying the precision of measurements. For example a 2 decimal place balance is precise to ±0.005 g.

Percentage errors – using a 2 decimal place balance, the errors when weighing 0.1 g and 0.01 g are identical yet the overall percentage errors are ±10% and 100% respectively (including the precision of the zero readings).

Overall percentage error – this arises when several measurements are used to achieve an overall result. It is approximately equal to the sum of the percentage errors in the individual readings although there are more complex treatments.
– In a simple weighing the overall percentage error is based on two readings, a zero or tare and the mass of the sample.
– In a titration the overall percentage instrumental error might be the sum of the percentage errors from the weighing, the volumetric flask, the pipette, the burette and the standard solution.

Reliability – this is assessed through comparison of an individual result with a reference or class mean.

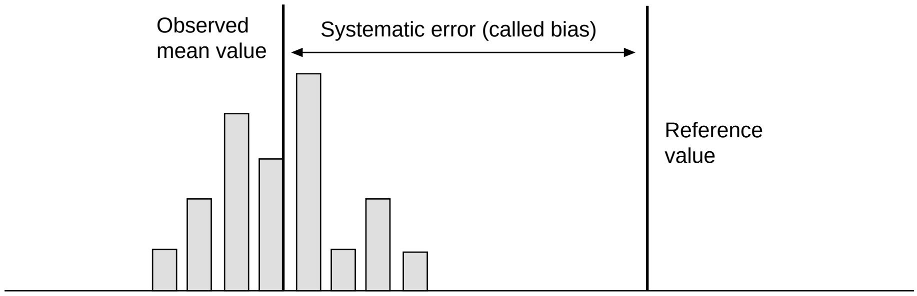
Assessing the reliability of an individual result allows a judgement to be made regarding the level of mistakes. The overall percentage error based on instrumental readings is unavoidable whereas mistakes (human errors) are avoidable through repeating and reproducing the results.

Anomalous point – a data point that does not fit the pattern of the graph.

Trueness

Trueness is defined as: The difference between the observed mean value and the reference value.

A true value is never achievable because there is always some random variation and it is recommended that the indefinite article is always used *ie* 'a' and not 'the'.



Bias is a measure of trueness.
Accuracy comprises bias and precision.

Figure 3 Accuracy, precision and trueness

Teaching tips

Introducing the vocabulary to the class
Professional judgement should be used here to decide how many terms it is appropriate to introduce to each class. The list of definitions given above is meant to be a comprehensive list to aid teaching at all levels.

There are several ways of introducing the terms to the class:

- List some of the words on the board and ask the students to write down the meanings. Discuss their answers.
- Use everyday ideas to introduce the terms and promote discussion *eg*
  – Accuracy and precision are required to succeed at darts and archery.
  – A cookery book must contain recipes that are repeatable and reproducible, otherwise no one would want to buy it.
- Using the student worksheet **Bulls eye to win**. This could also be used as a homework exercise.
- Analysis and interpretation of experimental data. The class could be presented with a set of results if there is not enough time to carry out the weighing experiment first.

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The weighing experiment (method 1)

- Set up the balances at different places in the room.
- Divide the class into groups.
- Present the class with some common identical objects – *eg* Mars® Bars.
- Each group weighs several Mars® Bars, using one weighing device.
- They then record the results and the weighing device used.
- They work out the average mass of a Mars® Bar.
- Add the result to the class table or graph (on the board or OHP).
- Plot the class results (from the table) individually.

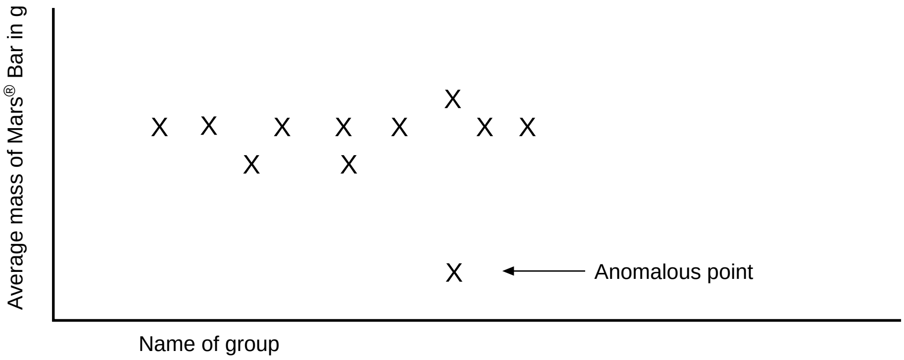

Figure 4 Sketch graph showing simple results

Interpreting the results
The results show a scatter close to a true value of the object (see Figure 4). You need to draw in the line of best fit and highlight any anomalous results. This approach is built on the assumption that all the Mars® Bars are the same mass, and that balances are the only variable. Any difference in the mass of the actual Mars® Bars will be minimised, because each group has used the average mass of several Mars® Bars in their result.

The weighing experiment (method 2)

As homework, ask the class to weigh the same object, such as a Mars® Bar, three times on their kitchen scales.

- Record the mass of the object each time they weigh it.
- Work out the average mass.
- Bring the object into school.
- Reweigh the object using a balance at school.
- Add the result to the class table or graph (on the board or OHP).
- Plot the class results (from the table) individually.

Interpreting the results
The results should show a scatter close to the declared value of the mass of the object. Manufacturers are given a tolerance on their products and therefore there is no true value or reference value. (Refer to notes in background information.) There will be a scatter of results, because in practice not all Mars® Bars will weigh exactly the same. You need to draw in the declared value (on the packet), allowed variability and highlight any anomalous results as shown in Figure 5.

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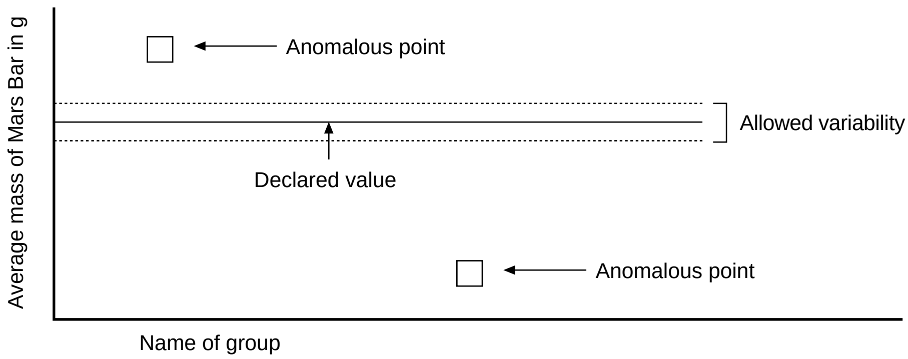

Figure 5 Sketch graph showing results, allowing for the variability in the mass of the Mars® Bar

The graph should be discussed either in groups or as a whole class. This could then be followed up by giving the students the worksheet **Interpreting the weighing experiment** either as a class exercise or a homework exercise.

Calibrating equipment

This should take the form of a teacher demonstration and discussion, and could be used in the final stage of the lesson. Some teachers may want their students to carry this out for themselves because it really depends on the quantity and quality of your balances. If available you could use a range of balances that read to a different number of decimal places. You may wish to demonstrate this to small groups at a time.

The class weighing exercise above should have already highlighted the fact that not all balances give an accurate reading, even if the readings are precise. It should be pointed out that from time to time, all balances need calibrating or resetting to make sure that they do give an accurate answer. An analytical chemist, carrying out measurements on a microscale (*eg* weighing to 0.001 mg) may have to calibrate the balance each time they use it. Even airflow can upset very sensitive balances as can the temperature of the object being weighed. The level of accuracy required is usually dictated by the application.

Resources

Classwork

- Mars® Bars or another product to weigh
- Kitchen scales (different types if available)
- Top pan balances of differing sensitivities
- Graph paper or graph plotting software or OHP.
- Student worksheets
  – Bulls eye to win
  – Interpreting the weighing experiment (method 1)
  – Interpreting the weighing experiment (method 2)

Demonstration

- Object of known mass such as a 10 g or 100 g weight
- 3 balances (if possible of different sensitivities). The least sensitive balance should not be calibrated correctly, as this will be done during the lesson.

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Practical tips

Class experiment
You may wish to add in an anomalous result, which could be used to discuss systematic errors later in the lesson. This could also save students from embarrassment, if there was only one other anomalous result. But in practice kitchen scales often provide the anomalous results. This could be checked before the lesson.

Demonstration

1. Using an object of known mass such as a 10 g or 100 g weight, place it in different positions on the most sensitive balance you have and record the results on the board. Dependent on the balance the reading will vary. How close is the reading to a 'true' value?

   Potential problem – make sure that you know a true value of your object at the start of the demonstration. Do not assume a 10 g weight has a mass of 10 g without checking it. It could be 10.1 g or 9.9 g.
2. Place the weight on a second balance (of different sensitivity). Do you get the same result?
3. Place the weight on a third balance and reweigh it. This time make sure that the balance is not zeroed correctly. If the balance has a setting-up levelling bubble, it should be off centre. The reading should be quite a bit out.
4. Show the group how to centre the bubble and carefully adjust the balance.
5. Re-weigh the object. It should now indicate the correct mass.
6. Stress the importance of correct setting up of equipment and of calibrating equipment, especially if accurate readings are required.
7. This leads naturally into a discussion about situations when accuracy is important and when it is not important.
8. The student worksheet **Does being accurate really matter?** could provide extension work or a follow up piece of homework.

Timing

2 hours (divided between classwork and homework)

Adapting resources

The student sheet **Bulls eye to win** could be adapted to meet the needs of the less able by turning question 3 into a cut and stick exercise and omitting questions 4 and 5.

Opportunities for other key skills

- Application of number

Background information

Industrial trading standards
Industry must comply to the Weights and Measures Act of 1985 and the Weights and Measures (Packaged Goods) Regulations 1986.

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In general terms the Act states: Goods which are sold in packages by weight or measure can be packed either to minimum quantity or to average quantity.[2]

For the minimum quantity each pack must contain at least the quantity marked on the pack (the nominal quantity). If equipment is used to make up the packs then the equipment must be tested and approved for trade use. The equipment does not have to be used, but if the quantity is estimated incorrectly, then the industry will have no defence.

For average quantity there are certain rules which must be followed called the Packers' Rules which are regularly monitored.

Packers' Rules

- The average content of the group must on average be at least the nominal quantity.
- No more than 2.5% (1 in 40) of the group may be non-standard *ie* (the nominal quantity) – (tolerable negative error).
- No package in the group may be inadequate *ie* (the nominal quantity) – 2 (tolerable negative error).

The tolerable negative error (TNE) is dependent on the nominal quantity.

Nominal quantity g or cm^3	**Tolerable negative error**
5–50	9% of nominal quantity
50–100	4.5 g or cm^3
100–200	4.5% of nominal quantity
200–300	9 g or cm^3
300–500	3.5% of nominal quantity
500–1000	15 g or cm^3
1000–10000	1.5% of nominal quantity
10000–150000	150 g or cm^3
Above 15000	1% of nominal quantity

Table 1 Legal industrial tolerance levels

The mass of the Mars® Bar

Teachers need to be aware of the common use of the word 'weigh' to determine mass. The average mass of a batch of products should be no less than the declared mass of 65 g.

The mass stated on the pack does not include the mass of the packaging.

Millions of Mars® Bars are produced daily. The few Mars® Bars weighed in class may not be representative of the batch.

Confectionery items weighing less than 50 g are not legally required to show a mass on the wrapper and this exemption applies to both standard and promotion packs.

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Smarties and Milky Bar Buttons
These are packed by mass rather than by the number of sweets. The manufacturing process is a fast and highly automated one and it is impossible to pack exactly the same quantity into each pack. As with any average there will be some packs with above average mass and some below. All packs will, however, be above an agreed minimum level.

Sources of information

http://www.tradingstandards.gov.uk/ (accessed June 2001)

Answers

Bulls eye to win

1. Jamal
2. The bulls eye
3. a) David, b) Helen, c) Jamal, d) Marie
4. David
5. Try an aim a bit lower and further over towards the right.

The weighing experiment (method 1)

1. Read off the value of the best fit line.
2. Use the value on the packet.

3 & 4 Accept sensible answers.

5. Mass of object (from graph) +/– difference (calculated in 4).
6. Accuracy of scales, precision of experiment, age of the product. Remember the manufacturer is allowed some variability.
7. See graph.
8. Dependent on results.
9. The balance has an error or the average was worked out incorrectly, or the individual results were not precise.
10. Accuracy is important; the customer should be getting the amount of product they are paying for.

The weighing experiment (method 2)
Questions 1–7 as method 1 above

8. To obtain the raw data of the anomalous result, go to the graph and find out whose measurement is incorrect. Go back to their exercise book and check the original readings. Write these readings up on the board. This should show if:
   (a) the average has been worked out incorrectly
   (b) if the results are not very precise
   (c) if there is a systematic error in the scales.
9. Dependent on results.

10&11 Refer to the raw data of the anomalous result.

12. Dependent on results.

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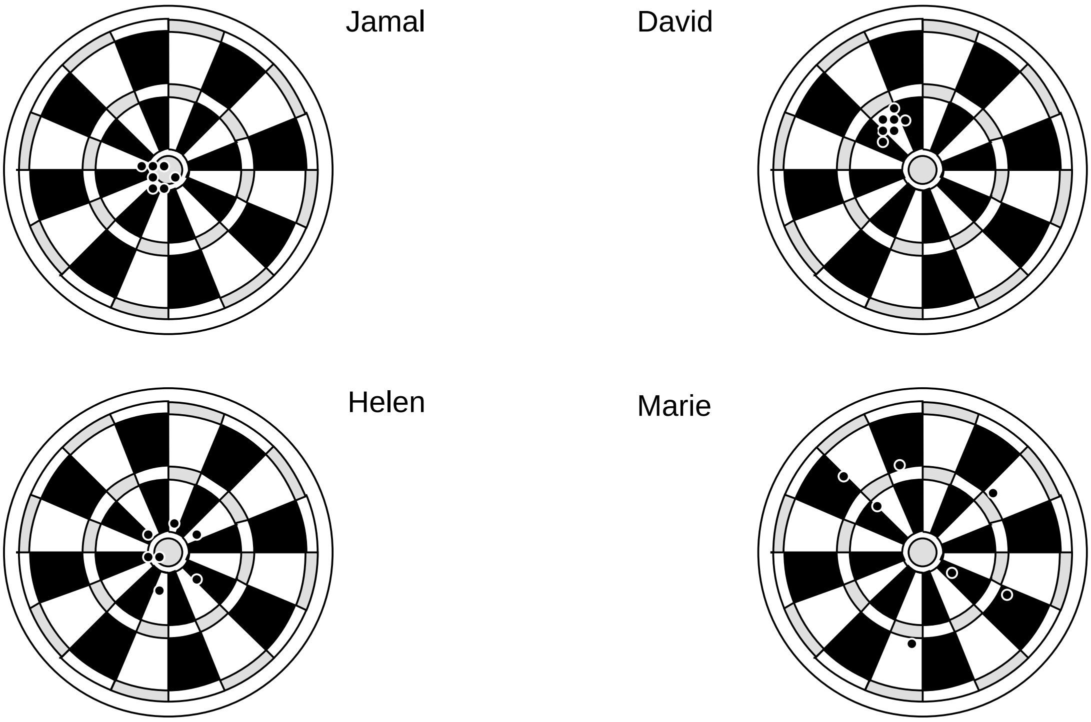
13. See reasons given under 8.

14. Make sure that you know how to work out averages, each time the object is weighed make sure that the balance reads zero and that nothing has been spilt on it *etc.*

Bulls eye to win

Jamal, David, Marie and Helen spent the afternoon playing darts. In the last round they set the target as the bulls eye. Each person was allowed seven throws. The results of their game are shown below.



Questions

1. Who won the game of darts?
2. What did the players choose to have as their reference value?
3. Whose game would you describe as:
   (a) Precise but inaccurate
   (b) Imprecise but accurate
   (c) Precise and accurate
   (d) Imprecise and inaccurate?
4. Who do you think needs to improve their game to avoid experiencing the same systematic error next time they play?
5. What advice would you give that person?

Interpreting the weighing experiment (method 1)

1. From the graph produced what is the mass of the Mars® Bar?

2. What is the expected value of the mass of the Mars® Bar? (Look at the wrapper.)

3. Is there a difference between the measured value and the expected value?

4. What is the size of the difference?

5. How would you write down the mass to include this error?

6. How can you explain the difference in results?

7. Are there any anomalous results?

8. Are the anomalous results showing a systematic or random error?

9. Suggest a reason to explain the anomalous result.

10. Discuss the importance of accuracy in the manufacture of Mars® Bars.

Interpreting the weighing experiment (method 2)

1. From the graph what is the mass of the object?

2. What is the expected value of the mass of the object? (Look at the wrapper.)

3. Is there a difference between the measured value and the expected value?

4. What is the size of the difference?

5. How would you write down the mass to include this error?

6. How can you explain the difference in results?

7. Are there any anomalous results?

8. If the answer is yes to question 7, check and record the raw data of the anomalous result.

9. Are the anomalous results showing a systematic or random error?

10. If the precision is poor, is it an example of poor repeatability or poor reproducibility?

11. If the results are biased (systematic error) what could this cause?

12. For the anomalous result, is there a difference between the home result and the school result? If so, what?

13. Suggest a reason to explain the anomalous result.

14. Suggest how this problem could be avoided in the future.

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Section 2: Choosing and using the right equipment

Teachers' notes

Many students have difficulty choosing the best apparatus to carry out the experiments and investigations. The worksheet **Choosing and using equipment** is designed to help in this.

Teaching tips

For lower ability students, supply each group with the actual equipment. The students should then be able to experience the different scales. For example, they could fill each measuring cylinder or beaker with 30 cm^3 of water, and use their observations to help decide upon the correct piece of equipment.

For more able students this activity could be introduced in class and then carried out as a homework exercise.

Resources

- Student worksheet
  – Choosing and using equipment

Timing

60 minutes or one lesson

Answers

1. At this stage the student is not expected to give the size of the beaker, measuring cylinder or thermometer, however they should include them in a list or a labelled diagram showing a heatproof mat, Bunsen burner, tripod and gauze.

2. C = 50 cm^3, there is no 30 mark on A, B is too small, the divisions on the scale on D go up in 10s whereas they go up in 5s on C. So more accurate to use C.

3. C. Water is usually heated in beakers. B and D are not beakers. More water will evaporate away if A is used as the water has a much larger surface area.

4. A or E. B and D do not go up to 100 °C, and C will not be so accurate as the same size thermometer goes up to 200 °C. E may or may not be the most accurate/easy to use.

5. Group 1 = 90.5 °C, group 2 = 98.6 °C, group 3 = 99.5 °C, group 4 = 101.2 °C

6. Group 3

7. Group 3

8. Groups 1 or 3

9. The result is much lower than expected.

10. Any 2 of:
- **(a)** James could have used the wrong sample of liquid.
- **(b)** His technique may not be very good. He may be taking the thermometer out of the water to read the scale.
- **(c)** The thermometer could have a systematic error.

11. The appropriate pair of answers from:
- **(a)** Repeat using the correct water supply.
- **(b)** Repeat, making sure that the thermometer is not taken out of the water.
- **(c)** Repeat, using a different thermometer, (or borrow results from group 3).

Choosing and using equipment

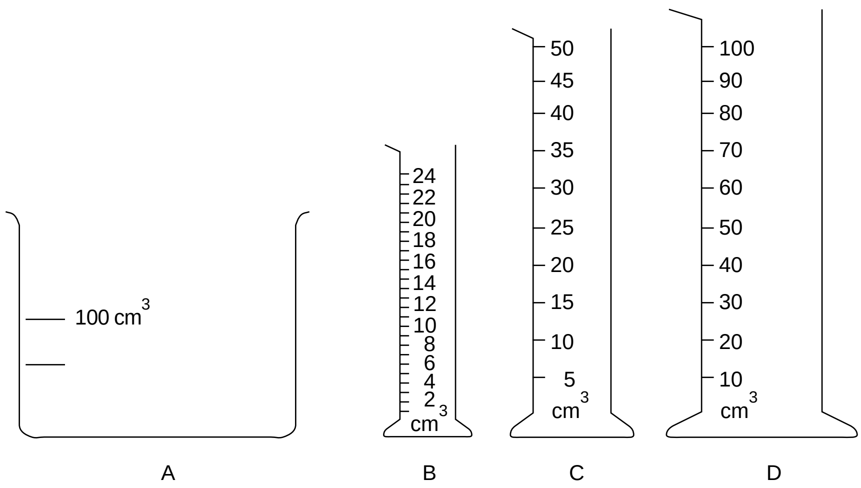
As part of a class investigation, James has been asked to heat 30 cm^3 of water and measure the temperature at which it boils. From his knowledge of science, he knows that water boils at 100 °C at a pressure of 1 atmosphere.

When James opened the cupboard to get out the equipment, he was very surprised to see so many different sized beakers, conical flasks, measuring cylinders and even thermometers. 'What should I use to get the most accurate result?' thought James.

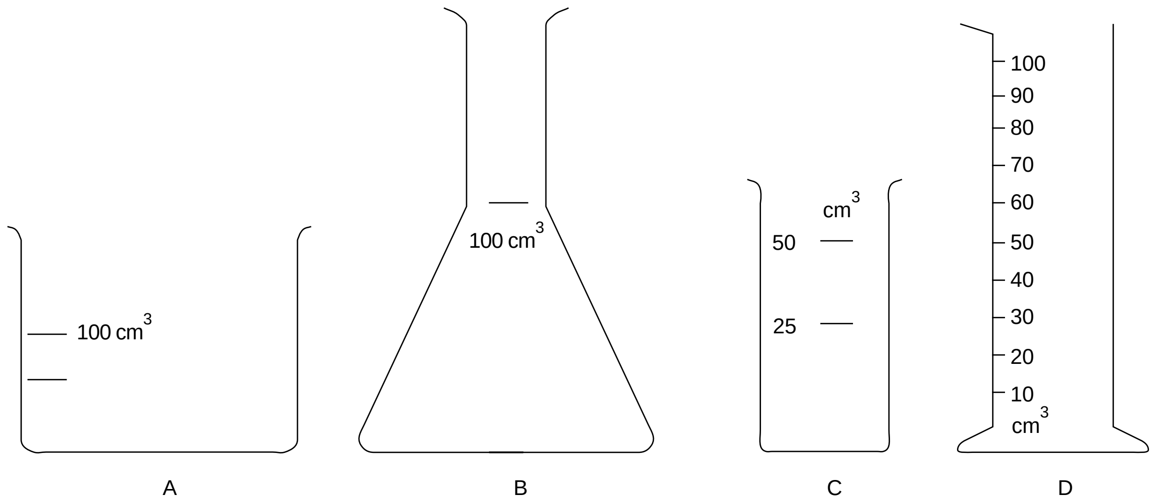
After choosing the equipment and taking some water from the container labelled 'distilled water', he carried out the experiment and noted down the result. However, he was not happy with the first result, so he repeated the experiment. He recorded his results in a table.

Your job is to help James choose the correct equipment.

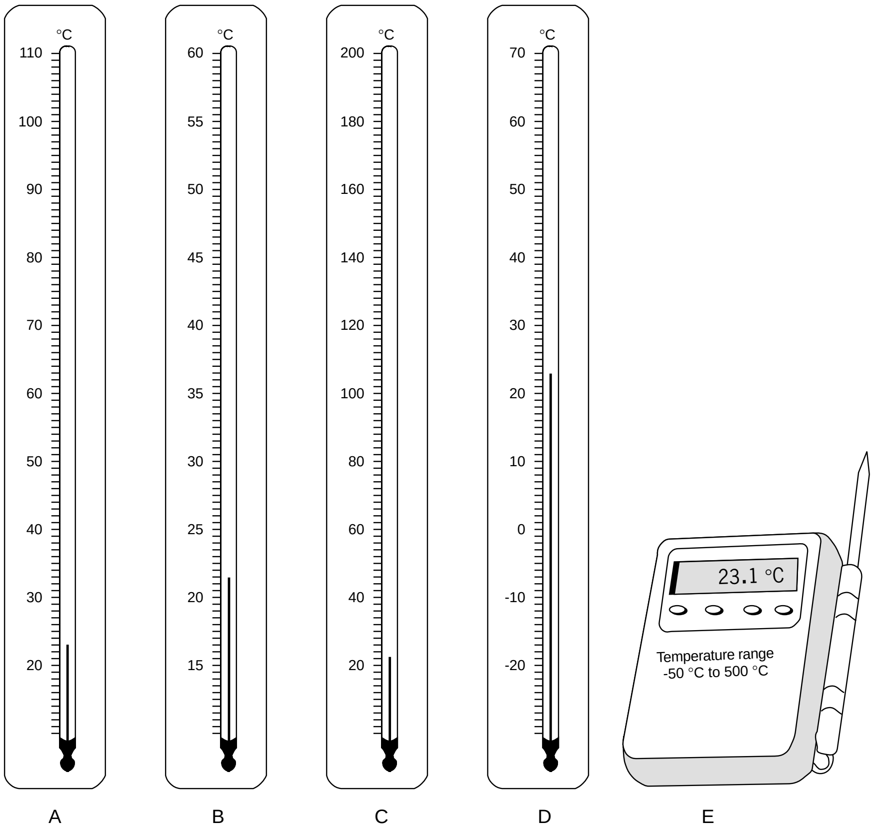
1. List all the pieces of equipment James will need to use to boil 30 cm^3 of water.
2. Which of the following containers should he use to measure out the water? Give a reason to support your answer.



3. Which of the following containers should he use to heat up the water? Give a reason to support your answer.



4. Which of the following thermometers should he use to measure the temperature of the water?
Give a reason to support your answer.



The table shown below shows the results of the groups in James' class.

	Boiling temperature / °C			
Test	**Group 1 & James**	**Group 2**	**Group 3**	**Group 4**
1	90.0	95.0	99.0	102.5
2	91.0	100.0	100.0	100.0
3	90.5	101.0	99.5	101
Average				

5. Work out the average temperature for each set of results.

6. Which group has measured an average boiling temperature closest to the expected value, if the pressure is 1 atmosphere?

7. Which group has the most accurate result?

8. Which group has the most precise readings?

9. Why do you think James is not happy with his group's result?

10. Suggest two reasons why James' results are different to the rest of the class.

11. Suggest how James could test out two reasons.

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Section 3: Does being accurate really matter?

Background information

This activity is intended to help pupils appreciate that different situations in science and everyday life require different degrees of accuracy and therefore different instruments to carry out the measurements. For example, if someone were intending to break the world record in the 5000 m run, they would need a stopwatch reading to 0.01 s during training, but if they just wanted to keep fit a normal wristwatch would do.

During accurate work, there is a need for instrument calibration. Precise results do not necessarily mean accurate results, if there is a systematic error. In some situations a systematic error will be negligible with no overall real effect, but in others it could be the difference between life and death if for example, the pharmacist's balance has a 15% error. It could become very expensive if the balance used to weigh gold ingots was incorrect while, if quality control in Mars® bar manufacture had a systematic error we might be lucky and get a bigger Mars® bar.

It should also be stressed that a systematic error is not important if the weighing is done by difference.

Teaching tips

Introduce the lesson using everyday examples of when measurements need to be accurate. You could go through some of the examples on the worksheet to get the class started. This lesson is suited to group work or individual work. Some of the answers to question 1 are totally dependent on context.

Resources

- Student worksheet
  – Does being accurate really matter?

Timing

60–70 minutes

Adapting resources

This worksheet can be adapted to meet the needs of the less able or younger student by:

- Reducing the number of statements on the sheet.
- Enlarging the statements in question 1, photocopying them on to card and then cutting out the statements so that the students can physically sort them into two groups.
- Replacing the written questions with pictures and then proceeding as above.
- Using a cut and stick approach to question 1 to minimise the writing.
- Reduce the difficulty of question 2, by only asking for one variable to be found.
- Removing questions 3 and 4 which are more demanding.

This worksheet can be adapted to meet the needs of the more able or older student by:

- Asking the students to add in some more situations.

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- Add the following to question 4. Suggest the degree of precision the instrument should have.
- Giving the student some numerical examples to work with *eg*
  - If a premature baby weighing 2 kg loses 0.5 kg; what is the overall percentage reduction in body mass?
  - If an adult man weighing 80 kg loses 0.5 kg; what is the overall percentage reduction in body mass?
  - Using your answers explain to what degree of accuracy you would measure the mass of the premature baby and the adult man.

Opportunities for other key skills

- Application of number
- Problem solving
- Working together.

Answers

1. Note that two of the measurements may be put into both categories.

Accurate measurements	**Rough estimations**
Timing 100 m breast stroke race.	Making a drink of squash.
Weighing a premature baby – a mass loss of 500 g could be vital. **(C)**	Weighing an adult man. A measurement to the nearest kg will do.
Running in an international 800 m race.	Going for a 3 km jog each morning – it doesn't matter how far you go or how long you take as it is only for fitness.
Training for a 800 m race – practice of the real thing.	Training for a 800 m race – different distances may be run to gain general fitness etc.
Building an Olympic swimming pool – if the length is slightly wrong (even by 1 cm) then new Olympic or World records cannot be set. **(C)**	Measuring out sand, cement and gravel for concrete – this is done by parts, where a part could be a bucket full or a shovel.
Weighing out aspirin to make it into 500 mg tablets – a small difference in weight could have a dramatic effect on the patient. The tolerance level is +/– 15%. **(A)**	Measuring the air temperature.
Analysis of water – the volume of water must be known accurately so that the concentration of the pollutants can be measured. **(B)**	
Weighing an adult man if he is a boxer or jockey.	
In an orienteering competition, the direction is measured using a compass.	In an orienteering competition, the distance is often estimated by pacing.
The body temperature of a child with a fever. **(B)**	

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2.

	Variable 1	**Variable 2**
100 m breast stroke	Timing **(C)**	100 m length **(C)**
Running a 800 m race	Timing **(C)**	800 m distance **(C)**
Training for a 800 m race	Timing **(A)**	Distance **(A)**
Orienteering competition	Direction **(A)**	Distance **(A)**

All the other situations only have one variable that needs measuring.

3. See letters in bold, in the above tables.

4. Any acceptable instruments. In this answer it is important for the degree of precision to be given with the measuring instrument.

Does being accurate really matter?

Measuring out orange squash to make a drink of orange

Weighing an adult man

Timing a 100 m breast stroke swimming race

Weighing a premature baby

Going for a 3 km jog each morning in order to keep fit

You are on a building site and are asked to measure out some sand, cement and gravel, which will then be mixed together to make some concrete

Training for an 800 m running race

Running in an international 800 m race

The body temperature of a child with fever

You are building an Olympic swimming pool. You need to measure the length of the pool which is 50 m

You are taking part in an orienteering competition. In order to find the next marker you will need to measure the distance and direction of your next move

Analysis of water to see what pollutants are present. You will need to measure a known volume of water in order to carry out the analysis

Weighing out aspirin to make it into a 500 mg tablet

To help you decide what clothes to wear you measure the air temperature

1. Read the situations and sort them into two groups. The first group requires accurate measurements. In the second group a rougher estimation will be good enough. Present your answer in a table.
2. For each of the situations that require accurate measurements, decide if there is more than one variable that needs to be accurately measured. Record your results in a table. The first one has been done for you.

	Variable 1	**Variable 2**
100 m breast stroke	Timing	100 m length

3. For each of the situations that require accurate measurements decide how accurate each reading must be? Use the code A – accurate to 1 unit, B – accurate to 1/10 of a unit and C – accurate to 1/100 of a unit and add these letters to your table.
4. Name the type of instrument that you would use to make each measurement and give its precision.