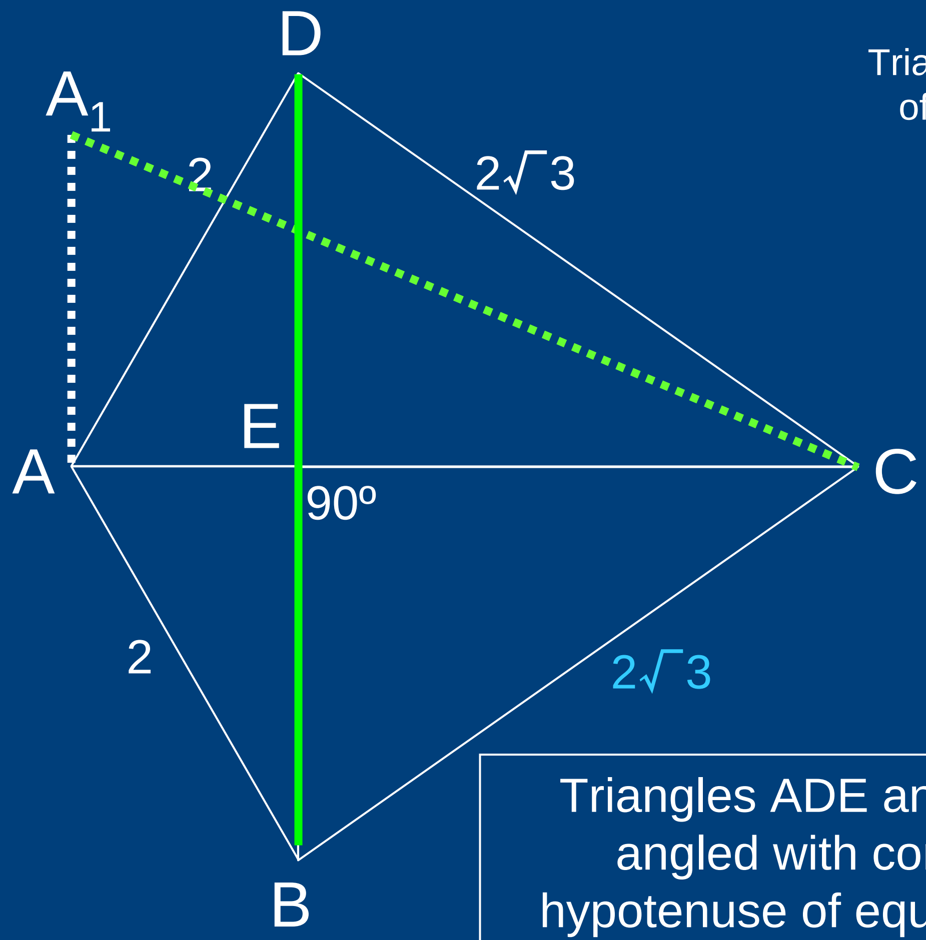
RSC ‘Chinese Puzzle’ competition – solution

April 2007

- Following slides are not a highly formalised proof, but show schematically how dimensions and angles are derived (indicated in blue) based on initial data (white).
- Calculations are made simpler by there being a number of right-angled triangles with angles 30º, 60º and 90º, noting that sin 30º = ½ and sin 60º = $\frac{1}{2}\sqrt{3}$.
- Pythagoras’ Theorem is used extensively in this three-dimensional geometry problem.
- Methods of solution provided in the competition vary in style, but those capturing the fundamentals and final answers shown here have been deemed to be correct.

Part I – proof of symmetry and angle between BD and A_1C

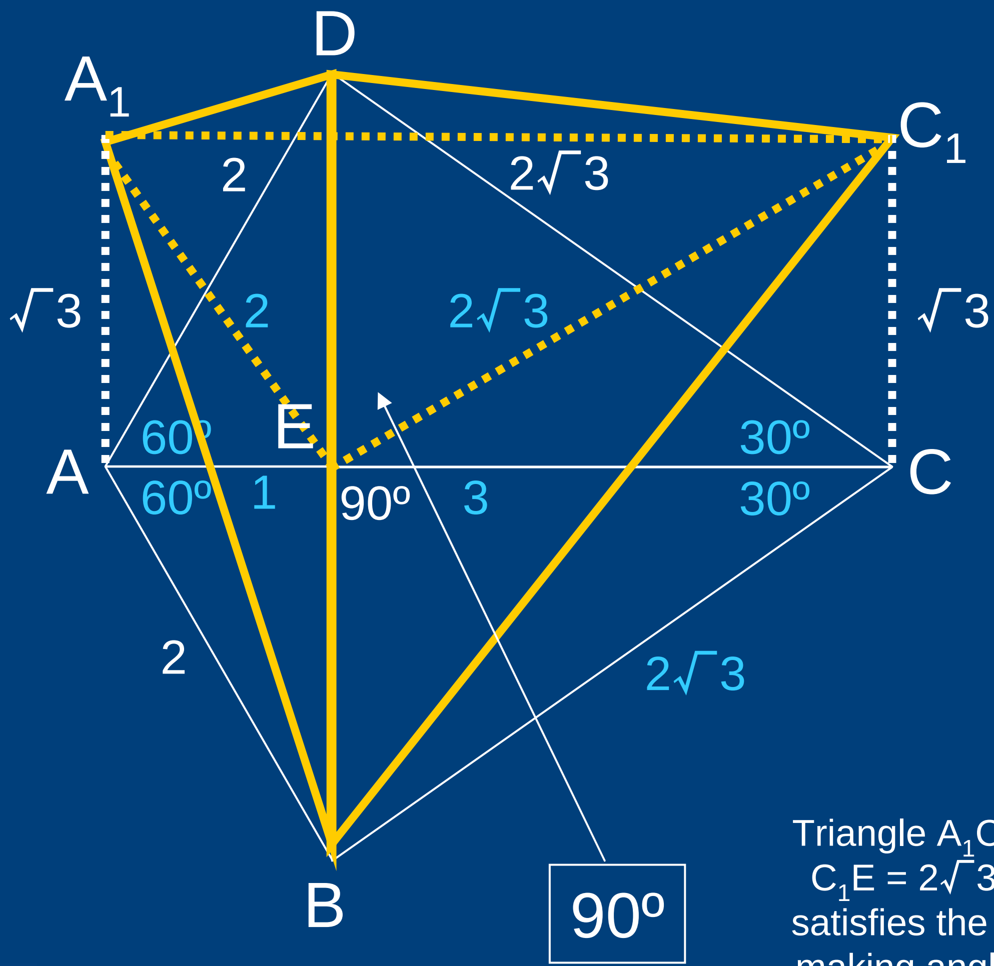


Triangle AA_1C is coming out of the plane of the paper

Triangles ADE and ABE are both right-angled with common side AE and hypotenuse of equal length 2. Therefore, BE = DE and there must be symmetry about the plane of AA_1C, so that $BD \perp A_1C$

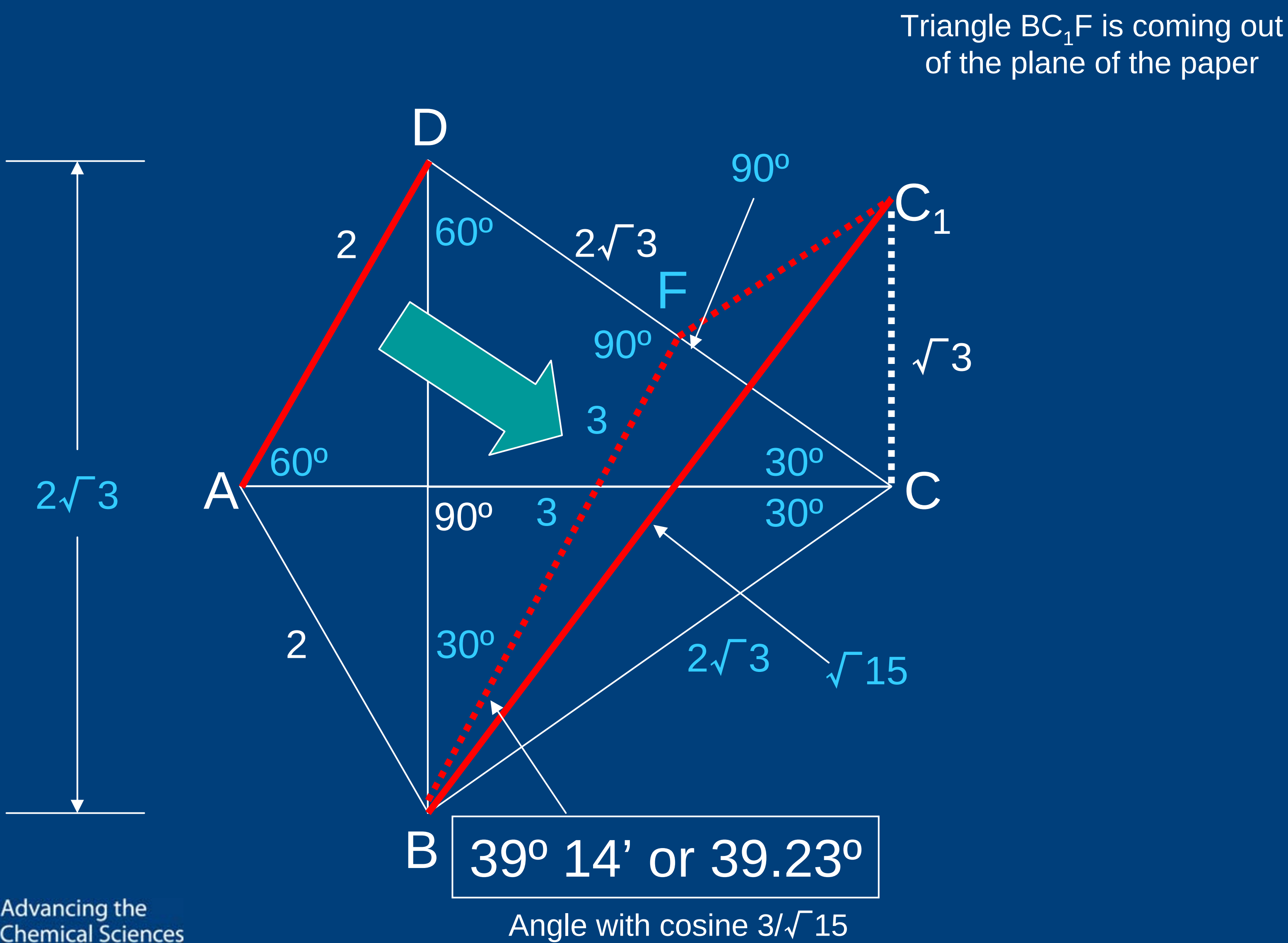
Part II – angle between planes

Triangles A_1BD and BC_1D are coming out of the plane of the paper



Triangle A_1C_1E has sides $A_1E = 2$, $C_1E = 2\sqrt{3}$ and $A_1C_1 = 4$, which satisfies the Pythagoras condition, making angle A_1EC_1 a right angle.

Part III – angle between lines



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