ELASTOCAPILLARY FLOW IN DEFORMABLE PDMS MICROCHANNELS

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ABSTRACT

We investigate the elastocapillary flow of Newtonian liquid in rectangular PDMS microchannels having a deformable membrane wall. The inward deflection of the membrane under negative capillary pressure is found to increase the filling speed in horizontal deformable microchannels, and the capillary rise in vertical deformable microchannels. A theoretical model is developed to quantitatively predict these effects, and a non-dimensional parameter $J$, which represents the ratio of capillary force to the mechanical restoring force, is seen to emerge as an important parameter in quantifying elastocapillary effects. The model predictions show good agreement with experimental data obtained from deformable channels fabricated in PDMS.

KEYWORDS: Elastocapillarity, Deformable wall, Filling speed, Rise height, PDMS microchannels

INTRODUCTION

The capillary flow of liquids can be successfully exploited to drive fluid flow in passive microfluidic devices. When liquid flows by capillary action in a microchannel with a deformable wall, this wall deflects inwards due to the Young-Laplace pressure drop across the curved liquid meniscus. As a result, the radius of curvature of the meniscus decreases, thereby increasing the negative capillary driving pressure and enhancing the capillary flow in such channels. In horizontal deformable microchannels, the effective Washburn coefficient is increased in comparison to an identical rigid channel. In vertical deformable channels, the capillary rise height is higher than that in an identical rigid channel. The increase in filling speed in horizontal silicon nanochannels with a flexible capping layer has been reported and studied using a simple mathematical model in [1].

THEORY

Consider a thin, rectangular polymer membrane of width $w$, length $L$ and thickness $t$, forming one wall of a rectangular microchannel as shown in Fig. (1). The liquid (gage) pressure $-p(x)$ acts transversely on the membrane, causing a deflection $\omega(x,y)$. Let $\sigma$ and $\theta$ be respectively the surface tension and contact angle of the wetting liquid, and $D$ be the flexural rigidity of the membrane wall. For typical channels having $w \ll L$, and built-in boundary conditions at the edges, it can be shown that the deflection is given by

$$\omega(x,y) = \delta(x) \left[ 4 \left( \frac{y}{w} \right)^2 - 1 \right]^2$$

(1)

where

$$\delta(x) = \omega(x,0) = -\frac{p(x)w^4}{384D}$$

(2)

is the maximum deflection at a given $x$, which occurs at the centreline $y = 0$.

The pressure $-p(x)$ acting on the membrane is maximum just behind the meniscus, and decreases in magnitude further upstream along the channel. Downstream of the meniscus, the channel is still occupied by air at ambient pressure and the load impressed on the membrane drops to zero. The maximum deflection $\hat{\delta}$ occurs at a location $x = \hat{x}$, a short distance behind the meniscus at $x = \hat{x}'$, as shown in Fig.
Beyond $\dot{x}$, the membrane quickly relaxes back to zero deflection; our numerical simulations showed that $\delta^*=2\delta^*$ to a very good approximation, in agreement with the assumption in [1].

![Diagram](image)

**Figure 1: Capillary flow in a deformable channel and the coordinate system used for analysis.**

For channels of large aspect ratio $a = w/h >> 1$, we can then show that $\xi = 2\xi^* = 1 - \sqrt{1-J}$, where $\xi^* = \delta^*/h$, $\xi = \delta h$, and the dimensionless parameter $J = w^4 \sigma \cos \theta / 96 Dh^2$ is the ratio of the capillary force $2a\sigma \cos \theta \ dx$ to the mechanical restoring force at 50% membrane deflection $(192D/aw^2) \ dx$. When $J > 1$, the membrane is too compliant and collapses under capillary pressure. If we define the elastocapillary length $\lambda$ as $\sqrt{D/\sigma \cos \theta}$, then we may write $J = a^4 (h/\lambda)^2 / 96$.

Having computed the deflection of the membrane at the meniscus, we can now show that the relative increase in capillary rise height in a deformable vertical microchannel (as compared to a rigid channel of identical dimensions and surface properties) is given by

$$r = \frac{H_{\text{deformable}} - H_{\text{rigid}}}{H_{\text{rigid}}} = \frac{a}{1 + a} \frac{\xi}{2 - \xi} = \frac{a}{1 + a} \frac{1 - \sqrt{1-J}}{1 + \sqrt{1-J}}$$

Further, in deformable channels of large aspect ratio, the Washburn’s coefficient for the capillary filling of a newtonian liquid may be derived to be equal to

$$W_{\text{deformable}} = \frac{64D \left(1 - \frac{0.63}{a}\right) \xi \left(1 - \frac{4\xi^2}{5} + \frac{128\xi^2}{315} - \frac{256\xi^3}{3003}\right)}{\mu ha^4}$$

where $\mu$ is the viscosity of the wetting liquid. We get a higher capillary filling speed in comparison to a rigid channel when the parameter $J > 0.238$. Just prior to membrane collapse at $J = 1$, we obtain the maximum value of the Washburn’s coefficient in the deformable channel, about 19.2% higher than that in a rigid channel of similar dimensions.

**EXPERIMENTAL**

Experimental studies were performed on straight rectangular microchannels fabricated in PDMS by conventional soft lithography process [2]. The channels were closed on one side by a deformable membrane, 30 to 40 µm thick, also fabricated in PDMS by spin coating the prepolymer on a backing plate and curing. The channels so formed had a cross section of height 100 µm and widths ranging from 600 to 1300 µm. The filling speed and rise height of liquids were measured, respectively, in horizontally and
vertically mounted channels, using an experimental setup consisting of an optical microscope and a high speed camera.

**RESULTS AND DISCUSSION**

Significant improvement in Washburn’s coefficient in horizontal channels (reaching \(15.7 \pm 1.3\%\) at \(a = 11\)) and rise height in vertical channels (about \(32.5 \pm 1.2\%\) at \(a = 10\)) were observed during experiments due to the presence of the deformable wall. Theoretical predictions were found to be in good agreement with experimental data, as seen in Fig. (2).

![Figure 2](image-url)

*Figure 2: (a) Ratio of Washburn’s coefficient in a deformable channel to that in a rigid channel of same dimensions, plotted for various aspect ratios. Liquid used for experiments is propan-2-ol. Theoretical curve corresponds to \(h/\lambda = 0.0808\), with membrane collapse occurring at \(a = 11.01\). (b) Percent relative increase in rise height of propane-1,2-diol in a deformable channel, over that in a rigid channel, for various aspect ratios. Theoretical curve is for \(h/\lambda = 0.0856\) and membrane collapse is predicted at \(a = 10.7\)*

The inward deformation of the membrane was also separately confirmed by analyzing the intensity variation in fluorescence micrographs of Rhodamine-B dye solution filling the deformable PDMS micro-channels.

**CONCLUSION**

The theoretical model and experimental results show that the use of a deformable membrane as a channel wall is indeed a viable technique for enhancing capillary flow in passive microfluidic devices, and can potentially be used in Lab on a Chip or µ-TAS devices in which faster response and reduced cycle times are desired.

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**REFERENCES**


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