RSC ‘Chinese Puzzle’ competition – solution

April 2007

- Following slides are not a highly formalised proof, but show schematically how dimensions and angles are derived (indicated in blue) based on initial data (white).
- Calculations are made simpler by there being a number of right-angled triangles with angles 30°, 60° and 90°, noting that sin 30° = ½ and sin 60° = ½√3.
- Pythagoras’ Theorem is used extensively in this three-dimensional geometry problem.
- Methods of solution provided in the competition vary in style, but those capturing the fundamentals and final answers shown here have been deemed to be correct.
Part I – proof of symmetry and angle between BD and $A_1C$

Triangles $ADE$ and $ABE$ are both right-angled with common side $AE$ and hypotenuse of equal length 2. Therefore, $BE = DE$ and there must be symmetry about the plane of $AA_1C$, so that $BD \perp A_1C$.
Part II – angle between planes

Triangles $A_1BD$ and $BC_1D$ are coming out of the plane of the paper.

Triangle $A_1C_1E$ has sides $A_1E = 2$, $C_1E = 2\sqrt{3}$ and $A_1C_1 = 4$, which satisfies the Pythagoras condition, making angle $A_1EC_1$ a right angle.
Part III – angle between lines

Triangle BC,F is coming out of the plane of the paper

39° 14’ or 39.23°

Angle with cosine $3/\sqrt{15}$