ONE DIMENSIONAL MODEL OF THERMORESISTIVE MICRO CALORIMETRIC FLOW SENSORS FOR GASES AND LIQUIDS CONSIDERING PRANDTL NUMBER EFFECT

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ABSTRACT
To design an integrated thermoresistive micro calorimetric flow (TMCF) sensor for gases and liquids, it is essential to develop a compact analytical model as a function of Prandtl number (Pr). In this paper, we present a simple one-dimensional (1D) model of a thin film based TMCF sensor for different fluids. The proposed model, validated by the CFD simulations and experimental data, is used for systematically studying the effect of key design parameters on the sensor performance. The normalized 1D model can be applied to the system-level design of TMCF sensors for different types of fluids.

KEYWORDS: Micro calorimetric flow sensor, Thermoresistive, Prandtl number, 1D sensor model

INTRODUCTION
Micro thermal flow sensors show great applicability in medical ventilation, environmental monitoring and microfluidic devices due to the benefits of miniaturization, low power consumption and low cost, etc. Various micro thermal flow sensors have been investigated [1], but very few scaling studies of these sensors were reported [2]. To date, there is no simple and reliable 1D TMCF sensor model applicable for both gases and liquids. To address this issue, it is essential to develop a compact analytical model for TMCF sensors. Lammerink et al. [3] developed a sensor model for a free standing structure and Nguyen et al. [4] further advanced it to a nonintrusive membrane-mounted structure; however, both of them assumed that the temperature was equal to ambient temperature at an infinite distance from the micro heater. In reality, the thin film is usually supported by the massive silicon substrate, which can be regarded as a heat sink. Therefore, at the edge of thin film, one should handle the temperature of thin film equals to ambient temperature. Herein, based on the finite sensor geometry as shown in Figure 1, we propose a compact 1D theoretical model for a TCMF sensor under the open flow condition. With the validation of CFD simulations and experimental data, the proposed model is firstly used for systematic study of the effect of key design parameters on the sensor performance; then applied to different fluids flow studies with different Prandtl numbers, Pr defined as the ratio of momentum diffusivity to thermal diffusivity.

Figure 1: Schematics of a TMCF sensor at Constant Temperature (CT) Mode (top) and the 1D model for sensor output ΔT with input velocity U (bottom).

Figure 2: Comparison of the TMCF sensor output between the CFD model and the experimental results [6].
THEORY AND MODELING

As shown in Figure 1, a TCMF sensor usually consists of a micro-heater at the center and two symmetrically located upstream and downstream temperature sensors with the output of \( T_u \) and \( T_d \), respectively. In the absence of flow, the temperature profile is symmetrically distributed around the heater. While in the presence of flow, the differential temperature \( \Delta T \) of TCMF sensor between \( T_u \) and \( T_d \) can be related to micro convective heat transfer and the corresponding input flow velocity \( U \).

Both momentum and thermal boundary layer phenomena during the TCMF sensor’s operation are essential for successful modeling. At Constant Temperature (CT) mode, based on the assumptions of the linear velocity profile above the thin film, the average momentum boundary layer thickness \( \delta_m \) and thermal boundary layer thickness \( \delta_t \) [5], we can reduce the TCMF sensor’s nonlinear differential equation for temperature \( T(x) \), to a linear 1D model in response to the input velocity \( U \) (see Figure 1) as follows:

\[
(k_s t + \frac{1}{2} k_f \delta_t + \frac{1}{2} k_f h) \frac{d^2 T(x)}{dx^2} - \frac{\rho C_p U}{6\delta} \frac{dT(x)}{dx} \delta_t^2 - \left( \frac{k_f}{\delta_t} + \frac{k_f h}{h} \right) T(x) = 0
\]  

(1)

where \( h \) is the cavity height, \( t \) is the thin film thickness, \( k_s, k_f \) and \( k_f h \) are the thermal conductivity of thin film, moving fluid and fluid in the cavity, respectively. In addition, we constructed a numerical model in a commercial CFD code (CFDRC, ESI-CFD, USA) for a reported TCMF sensor [6]. As shown in Figure 2, the CFD model can successfully predict the fabricated TCMF sensor response \( \Delta T(U) \). To have more accurate prediction of \( \Delta T(U) \), time-consuming CFD simulations are necessary. Figure 3 shows the predicted sensor sensitivity and output as a function of input velocity for the flow range suitable for typical gas monitoring. Good agreement between the 1D model results and the CFD simulation results with a fitting factor of 4.5 is observed in Figure 3’s inset, which demonstrates that our 1D model can predict sensor response and save significant CPU time.

![Figure 3: The TCMF sensor’s sensitivity and output as a function of input velocity \( U \) and Re. \( (2S=1500\mu m, 2L=600\mu m, 2W=100\mu m, D=120\mu m, h=150\mu m, t=4\mu m) \). Inset shows the comparison of the sensor output between CFD and 1D model.](image)

![Figure 4: The effect of the distance between the heater and temperature sensors (D) on the sensor output \( \Delta T \). Note: the parameters (S, L, W, h, t) are the same as Figure 3.](image)

RESULTS AND DISCUSSION

Figure 4 shows that an optimal distance \( (D/L=0.3–0.4) \) exists between the heater and temperature sensor to achieve the largest output. Meanwhile, one should mount the temperature sensors close to the heater to detect larger velocity; similar behavior was also observed in CFD results. Figure 5(a) makes clear that a thinner film design could provide a lower power and high sensitivity performance. It should be noted that the minimum thickness of thin film is indeed limited by the fabrication process and the required mechanical strength. Parametric study of the effect of cavity height on the sensor behavior (Figure 5(b)) reveals that there is an optimum height (more than 33µm) to achieve the best sensor performance.
Figure 5: The effects of (a) thin film thickness $t$ and (b) cavity height $h$ on the sensor power consumption and sensitivity. Note: $U=1m/s$ and the parameters ($S, L, W, D, h$) are the same as Figure 3.

With the help of Buckingham $\pi$ theorem, the sensor output can be normalized with respect to the heater temperature $T_h$ as a function of Reynolds number ($Re$) and Prandtl number ($Pr$) for both gases and liquids:

$$\Delta T / T_h = f\left(\phi, k_f^*\right); \phi = \sqrt{RePr^{1/3}}$$ (2)

Figure 6 shows the sensor output $\Delta T/T_h$ as a function of normalized input velocity $\phi$ for air, water and DNA solution (2.6µg/mL), demonstrating the 1D model can be applied to different types of fluids.

Figure 6: Normalized TMCF sensor output for air, water and DNA solution. Note: the parameters ($S, L, W, h, D, t$) are the same as Figure 3.

CONCLUSION

In conclusion, we proposed a general 1D model to predict the TMCF sensor response for both liquids and gases. It is very useful for the system-level design of next generation of TMCF sensors in medical ventilation devices and microfluidics.

REFERENCES


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