Appendix 1

1.1 Rescaled Range Analysis (R/S)

The R/S analysis was introduced by Hurst[1] and attempts to find patterns that might repeat in the future. The method employs two variables, the range, R and the standard deviation, S, of the data. According to the R/S method, a natural record in time, $X(N) = x(1), x(2), \ldots, x(N)$ is transformed into a new variable $y(n,N)$ in a certain time period n ($n = 1,2,\ldots,N$) from the average, $\langle x \rangle_N = \frac{1}{N}\sum_{n=1}^{N} x(n)$, over a period of N time units.[1] $y(n,N)$ is called accumulated departure of the natural record in time.[1] The transformation follows the formula:

$$y(n,N) = \sum_{i=1}^{n}(x(i) - \langle x \rangle_N) \tag{1.1}$$

The rescaled range is calculated from (1.2):[1–5]

$$R/S = \frac{R(n)}{S(n)} \tag{1.2}$$

The range $R(n)$ in (1.2) is defined as the distance between the minimum and maximum value of $y(n,N)$ by:

$$R(n) = \max_{1 \leqslant n \leqslant N} y(n,N) - \min_{1 \leqslant n \leqslant N} y(n,N) \tag{1.3}$$

The standard deviation $S(n)$ in (1.2) is calculated by :

$$S(n) = \sqrt{\frac{1}{N}\sum_{n=1}^{N}(x(n) - \langle x \rangle_N)^2} \tag{1.4}$$

R/S is expected to show a power-law dependence on the bin size n

$$\frac{R(n)}{S(n)} = C \cdot n^H \tag{1.5}$$

where H is the Hurst exponent and C is a proportionality constant.

The log transformation of the last equation is a linear relation

$$\log\left(\frac{R(n)}{S(n)}\right) = \log(C) + H \cdot \log(n) \tag{1.6}$$

from which,exponent H can be estimated as the slope of the best fit line.

1.2 The Roughness-Length Method (R-L)

The R-L method is based on the Fractal Geometry concept which is used for the accurate calculation of the Hurst Exponent.[4] The calculation of the Hurst exponent through the R-L method is performed by calculating the standard deviation $S(n)$ of the height values of a segment of length n of a self-affine profile by the formula:[3–5]

$$S(n) = A \cdot n^H \tag{1.7}$$

In (1.7) A is a proportionality constant that describes the profile waviness amplitude and H is the Hurst exponent. $S(n)$ is calculated by

$$S(n) = \frac{1}{\xi_n} \cdot \sum_{i=1}^{\xi_n} \sqrt{\frac{1}{m_i - 2}\sum_{j \in n_i}(x_j - \langle x \rangle_{n_i})^2} \tag{1.8}$$

where ξ_n is the total number of segments of width n in which the profile is divided, m_i is the number of points included in the i-th segment n_i, x_j is the aperture of the profile nodes from the best fit line and $\langle x \rangle_{n_i}$ is the mean value of x_j in the segment n_i. Representing the pairs of $\{n, S(n)\}$ in a double logarithmic diagram, the Hurst exponent is calculated through a least square fit.[4]

1.3 The Variogram Method

The variogram, also known as variance of the increments,[3] is the expected value of the squared difference between two x values in a trace separated by a distance h *i.e.* the sample variogram $2\gamma(n,h)$ of a series $x(n)$ is measured by the following equation[3–6]

$$2\gamma(n,h) = \frac{1}{M}\sum_{i=1}^{M}[x(n_i) - x(n_i + h)]^2 \tag{1.9}$$

where M is the total number of pairs of roughness heights of the profile that are spaced at a lag distance h.

The variogram $2\gamma(n,h)$ and the Hurst exponent H are related with the equation

$$2\gamma(n,h) = K \cdot h^{2H} \tag{1.10}$$

where K is a proportionality constant.[6] The slope of the linear fit of $\log(2\gamma(n,h))$ and $\log(h)$ equals $2H$.

References

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